1.7 The Chain Rule

Example 1 Find the derivative of $(x^2 - 5x + 1)^3$.

Here is the "hard" way to do it:

$$\frac{d}{dx}\left[(x^2 - 5x + 1)^3\right] = \frac{d}{dx}\left[x^6 - 15x^5 + 78x^4 - 155x^3 + 78x^2 - 15x + 1\right]$$
$$= 6x^5 - 75x^4 + 312x^3 - 465x^2 + 156x - 15$$
$$= \text{fancy factoring ....}$$
$$= 3(2x - 5)(x^2 - 5x + 1)^2$$

Now, the "easy" way:

The Extended Power Rule

$$\frac{d}{dx}\left[f(x)^k\right] = k\left[f(x)^{k-1}\right] \cdot f'(x)$$

Try Example 1 again:

Example 2 Find the derivative of: $f(x) = \sqrt{2x^3 - 4x + 1}$

Example 3 Find the derivative of: $y = (2x + 3)^4 (x^2 - 5)^7$

The Chain Rule

Let $y = f(g(x))$, where both $f(x)$ and $g(x)$ are both differentiable.

$$y' = \frac{d}{dx}[f(g(x))]$$
$$= f'(g(x)) \cdot g'(x)$$

Example 4 Find the derivative of: $f(x) = \sqrt[3]{\frac{2x+1}{x^2+5}}$

Example 5 If $f(x) = 3x^2 - 5x$ and $g(x) = x^2 + 1$ find $\frac{d}{dx}[f(g(x))]$

Alternate form of the Chain Rule

If $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example 6 Write the function $y = \sqrt{(x^2+3)^3 + 5} - 8$ as a composite function and find y'.