

Math 146 9.3 - Testing for Two Means - Paired Data (Dependent Samples)

When there is a natural pairing of data (before-after, pre test - post test, husband-wife, etc.) we can compare the difference of the means by first subtracting the data values from one sample from the paired data values of the second sample, and treating the difference as a single random sample.

NOTATION

- d = the difference of the two data values in a matched pair
 - μ_d = the mean value of the differences d for the population of matched pairs
 - \bar{d} = the mean of the difference of values for the matched pairs
 - s_d = sample standard deviation of the difference of the matched pairs
 - n = sample size
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REQUIREMENTS

1. Sample data are dependent, i.e., naturally matched pairs.
 2. The samples are simple random samples.
 3. Either $n > 30$, or the distribution of population differences is normal.
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TEST STATISTICS CONFIDENCE INTERVALS FOR PAIRED DATA

$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$ using a degrees of freedom $df = n - 1$. Typically $H_0 : \mu_d = 0$, but this is not necessary.

The confidence interval for μ_d is:

$$\bar{d} - E < \mu_d < \bar{d} + E \text{ where } E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

EXAMPLE 1 The following is a random sample of 12 students exam scores from Exam 2 and Exam 3 from a Math 099 course. Test the claim that individual student exam scores improve from Exam 2 to Exam 3, and find a confidence interval for the mean improvement. Use $\alpha = 0.05$. Be sure to test for normality.

Exam 2	39	46	35	40	39	51	36	31	29	33	34	51
Exam 3	40	48	42	45	39	47	39	33	42	45	39	47

EXAMPLE 2 An Outlier's Effect

Our bodies have a natural electrical field that is known to help heal wounds. Does changing the field strength slow healing? A series of experiments with newts investigated this question. In one experiment, the two hind limbs of 12 newts were assigned at random to either experimental or control groups. The electrical field in the experimental limbs was reduced to zero by applying a voltage. The control limbs were left alone. Here are the rates at which new cells closed a razor cut in each limb, in micrometers per hour.

Newt	1	2	3	4	5	6	7	8	9	10	11	12
Control Limb	36	41	39	42	44	39	39	56	33	20	49	30
Experimental Limb	28	31	27	33	33	38	45	25	28	33	47	23

- Is this a matched pairs experiment?
 - Find the difference in the matched pairs (control – experimental).
 - Make a stem and leaf plot of the differences and identify any outliers.
 - A good way to judge the effects of an outlier is to do your analysis twice, one with the outlier and one without. Carry out two t -tests to see if the mean healing rate is significantly lower in the experimental limbs, one test with all 12 data points, and a second test excluding the outlier. Does the outlier have a strong influence on your conclusion?
 - There are actually two outliers which can be found by creating a modified box-plot. Create a third test excluding both outliers and compare the P-value with the P-values from part (c). How do the results compare?
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