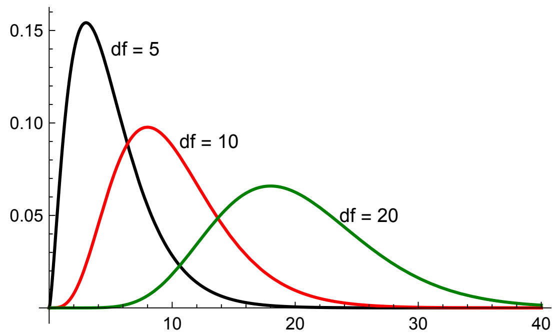
MATH 146 **7.3 - Estimating Variation; Variance and Standard Deviation**

When estimating variation, the **variance**, s^2, is the best point estimate for σ^2 (recall variance is an unbiased estimator.) In doing so, we use the **chi-square distribution** (pronounced kigh-square), whose distribution equation is

$$\chi^2 = \frac{(n-1)\,s^2}{\sigma^2}$$ where $n-1$ is the degrees of freedom.

Characteristics of the Chi-Square Distribution

- The degrees of freedom is the sample size minus one: df = $n-1$
- The distribution is not symmetric, but skewed right.
- The values of the distribution are never negative.
- In creating a confidence interval we need different left and right critical numbers: χ_L^2 and χ_R^2.



In constructing a confidence interval we need to

1) Insure the sample is a random sample from a normally distributed population.
2) Use the degrees of freedom $n-1$ and either use a table or technology to find χ_L^2 and χ_R^2.
3) The confidence interval for σ^2 is

$$\frac{(n-1)\,s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)\,s^2}{\chi_L^2}$$

4) To get a confidence interval for σ, take the square root of the left and right limits.

Example 1 The sample of foot lengths from our data collection survey ($n = 30$) had a sample variance of 18.99 in^2. Find a 95% confidence interval for the population variance and standard deviation.

Solution: Since $n = 30$, df = 29. Also, $\alpha/2 = 0.025$, and Table A-4 gives critical values using area *to the right* of the critical value. Using df = 29 and right-areas of 0.975 and 0.025 we get $\chi_L^2 = 16.047$, and $\chi_R^2 = 45.722$.

$$\frac{(n-1)\,s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)\,s^2}{\chi_L^2}$$

$$\frac{(29)\,(18.99)}{45.722} < \sigma^2 < \frac{(29)\,(18.99)}{16.047}$$

$$12.045 < \sigma^2 < 34.318$$

Also, an interval for σ is: $3.471 < \sigma < 5.858$.

Unfortunately the TI84 does not have a chi-square interval application, and the calculations need to be done as above.

Using **EasyCalc-Statistics**, we have:

Estimating Variance Using Statistics

Sample Variance=	18.9900
Sample Size=	30
Confidence Level=	0.95
Chi-Sq-Left CV=	16.0471
Chi-Sq-Right CV=	45.7223

95%-Conf Interval (Var)	
12.0447	34.3184

95%-Conf Interval (St.Dev.)	
3.4705	5.8582

EXAMPLE 2 The speed limit in front of a school zone is 25 mph, and the speed of ten cars were recorded:

23 24 22 20 27 24 25 21 23 24

Assuming the speeds are normally distributed (is this a valid assumption?) find a 90% confidence interval for the population variance and standard deviation.