MATH 146 **7.2 - ESTIMATING A POPULATION MEAN & THE *t*-DISTRIBUTION**

Just as we created confidence intervals for proportions using the characteristics of the standard normal curve, we also want to create distributions to estimate population means.

Creating Confidence Intervals for the Population Mean with σ KNOWN

To create a confidence interval for population means we need:

1) The population to be normally distributed, or
2) $n > 30$

Also, recall from the Central Limit Theorem, the standard deviation of the sample means is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. This means, if we know the population standard deviation, σ, the formula for the *margin of error* is

$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

EXAMPLE 1 Suppose the distribution of male heights has $\sigma = 2.8$ inches. A sample of 50 males had a sample mean of $\bar{x} = 69.2$ inches. Calculate a confidence interval to estimate the population mean height, μ, using $\alpha = 0.02$.

If the population mean isn't known, how likely is it to know the population standard deviation?

Creating Confidence Intervals for the Population Mean with σ NOT KNOWN

If the population standard deviation is not known, we use a t-distribution, which is very similar to the normal distribution:

1) It is bell-shaped
2) it is symmetric about the mean
3) the mean is 0

with these differences:

4) the standard deviation varies with sample size
5) the *t*-distribution is a family of curves depending on the degrees of freedom ($d.f. = n - 1$)
6) as the sample size increases, the *t*-distribution approaches the standard normal curve.

t-distribution -vs- standard normal

To create a confidence interval with a level α, the critical value is $t_{\alpha/2}$ instead of $z_{\alpha/2}$. The critical number $t_{\alpha/2}$ depends on both the confidence level, and the *degrees of freedom*, $df = n - 1$. We can find the critical number using Table A-3 or technology.

EXAMPLE 2 Find the critical number for $\alpha = 0.05$ and $n = 20$ using Table A-3 and invT(α/2,df).

Using the *t*-distribution for confidence intervals there are a few **requirements**:

1) The sample is a simple random sample
2) the population is normal **or** $n > 30$

Then, the confidence interval is $\bar{x} \pm E$, where $E = t_{\alpha/2} \frac{s}{\sqrt{n}}$ and the degrees of freedom is $df = n - 1$.

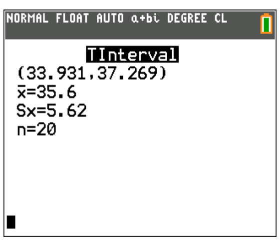
EXAMPLE 3 The speed limit in front of a school zone is 25 mph, and the speed of ten cars were recorded:

23 24 22 20 27 24 25 21 23 24

Assuming the speeds are normally distributed (is this a valid assumption?) find a 95% confidence interval for the mean speed.

EXAMPLE 4 An earlier study on the GPA of college students found the standard deviation to be 0.513 and a mean of 3.179. How many students would need to be surveyed to find a 97% confidence interval for the mean with an error of $E = 0.05$. Calculate the confidence interval.

EXAMPLE 5 I calculated a confidence interval on my calculator, but forgot what α-level I used. Can you find it?



EXAMPLE 6 Find a 95% confidence interval for the population mean using the following sample data set, and give a statement about the interval.

13 26 31 5 35 4 3 33 8 40 20