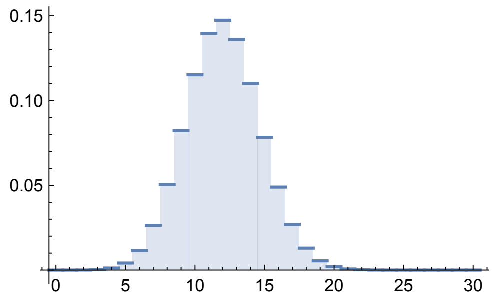
Math 146 6.6 - Approximating a Binomial Distribution with a Normal Distribution

Recall the binomial distribution back in section 5.2. The calculations are a bit daunting using

$$P(x) = \frac{n!}{x!\,(n-x)!}\, p^x\, q^{n-x}$$

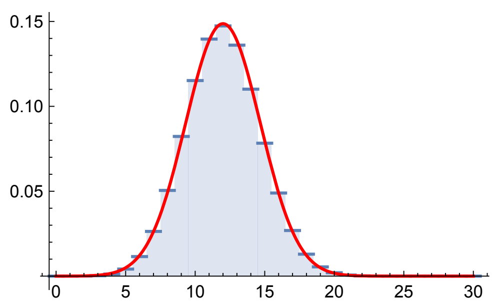
In many cases, a normal distribution can be used to approximate a binomial distributions. For example, consider the binomial distribution $B(30, 0.4)$, i.e., $n = 30$ and $p = 0.4$. The probability histogram for this distribution is



Note how similar it looks to a normal distribution, recall for a binomial distribution:

$$\mu = n\,p \quad \text{and } \sigma = \sqrt{n\,p\,q}$$

So, for a $B(30, 0.4)$ distribution we have $\mu = 12$ and $\sigma = 2.683$. A normal curve with this mean and standard deviation superimposed over the binomial histogram looks like:



Requirements to Approximate a Binomial with a Normal

Given a binomial distribution with sample size n, and probabilities p and $q = 1 - p$

- If $n\,p \geq 5$ and $n\,q \geq 5$ then the binomial distribution $B(n, p)$ can be reasonably by approximated by a normal distribution with $\mu = n\,p$ and $\sigma = \sqrt{n\,p\,q}$.

Continuity Correction

Looking at the normal curve and binomial plot above, you can see that the normal curve intersects each binomial column in the center. To approximate $P(5)$ with a normal curve we need to evaluate the normal distribution from 4.5 to 5.5, or from $x - 0.5$ to $x + 0.5$. This is called *the continuity correction.*

Example 1 For the binomial distribution $B(30, 0.4)$ Find:

a) $P(x = 10)$ using the binomial distribution, and

b) approximate $P(x = 10)$ using the normal distribution.

Example 2 For the the binomial distribution $B(50,\ 0.23)$ calculate $P(15 \le x \le 30)$ using the binomial distribution, and approximating it with a normal distribution.

Example 3 The probability of a baby being born a male is 0.512. In one thousand births, approximate the probability of having 520 or more males. Compare with the binomial calculation.

Example 4 For the binomial distribution $B\,(200,\ 0.05)$ approximate P_{95}.