

Math 146 5.3 — Poisson Distribution

The Poisson distribution is a discrete distribution, similar to the binomial, but with a few significant difference.

REQUIREMENTS FOR A POISSON DISTRIBUTION

1. The random variable x is the number of occurrences of an event over some interval. (*The interval can be time, distance, area, volume, etc.*)
2. The occurrences must be random.
3. The occurrences are independent
4. The occurrences are uniformly distributed over the interval being used.

CALCULATIONS OF POISSON PROBABILITIES

A Poisson distribution is only determined by one value, the mean μ .

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

where $e \approx 2.71828$, and μ is the mean of the distribution. Also, the standard deviation of a Poisson distribution is

$$\sigma = \sqrt{\mu}$$

On the TI-84 use the commands `poissonpdf(μ, x)` and `poissoncdf(μ, x)`

DIFFERENCES BETWEEN A BINOMIAL DISTRIBUTION AND A POISSON DISTRIBUTION

1. A binomial distribution is determined by the sample size n and the probability p ; a Poisson distribution is only determined by the mean μ .
2. In a binomial distribution, the possible values for the random variable x are $0, 1, 2, \dots, n$, while the random variable x in a Poisson distribution is $0, 1, 2, 3, \dots$ with no upper limit.

EXAMPLE 1 *The area around Mount St. Helens averages about 17 earthquakes per month.*

- (a) Find the probability of 15 earthquakes occurring in any particular month.
 - (b) Find the probability of at most 10 earthquakes occurring in any particular month.
 - (c) Find the probability of at least 20 earthquakes in any particular month.
 - (d) In December 2018, there were 80 earthquakes around St. Helens. Is this a rare event?
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EXAMPLE 2 The average number of horse fatalities per year at each four major horse tracks is 17.4. Santa Anita Park in Arcadia had 41 deaths last year.

- a) Find $P(x \leq 10)$
- b) Find $P(x = 41)$ and $P(x \geq 41)$
- c) Is having 41 horse fatalities unlikely? Unusual?

EXAMPLE 3 A single bank teller can help approximately 20 customers an hour. The mean number of customers a particular bank sees is 47.8 per hour.

- a) Find the probability of having few than 25 customers in an hour.
- b) Find the probability of having 60 customers during any given hour. This would require the availability of more than three tellers. Is this an unlikely event? An unusual event?