

## Math 146 5.2 - The Binomial Distribution

Suppose it is estimated that 2% of the population of a city in China has the Corona virus. If 50 people are randomly selected, what is the probability that exactly 3 are infected? What is the probability that at least 6 are infected?

### BINOMIAL DISTRIBUTION REQUIREMENTS

1. The procedure has a fixed number of trials.
2. The trials are independent\*. (Choosing one individual doesn't effect the probability of choosing another.)
3. Each trial has two outcomes classified into two categories (success and failure).
4. The probability of success is the same for each trial.

(\* If sample size is no more than 5% of the population we can always assume independence.)

### NOTATION

$n$  = the fixed number of trials

$p$  = the probability of success for one of the  $n$  trials, or  $P(S) = p$

$q$  = the probability of failure ( $q = 1 - p$ , or  $P(F) = q$ )

$x$  = the number of success in the  $n$  trials ( $0 \leq x \leq n$ )

### CALCULATING BINOMIAL PROBABILITIES

**Method I** (we will NOT use this method)

$$P(x) = \frac{n!}{(n-x)!x!} p^x q^{n-x} \text{ for } x = 0, 1, 2, \dots, n$$

**EXAMPLE 1** Suppose in a litter of 5 Labrador puppies there is a 35% chance that one is golden and 65% chance that it is black. Is this a binomial distribution?

Identify:  $S$ ,  $F$ ,  $n$ ,  $p$ , and  $q$ .

Find  $P(2)$ , that is the probability of getting exactly 2 golden labs.

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**Method II** - The TI-84

Pressing **2<sup>nd</sup> distr** (the vars button) gets us to the probability distribution menu.

Given:  $n = 5$  and  $p = 0.35$ , to find  $P(2)$  evaluate: `binomialpdf(n,p,x)`

(Note: Leaving out the  $x$  will give the entire probability distribution.)

**Method III** (Table A-1, we will NOT use this method)

**EXAMPLE 2** In packaging eggs, it is found that an egg has a probability of 10% of being damaged. The company packages eggs in cartons of 12 (1 dozen) and analyzes the probability of getting damaged eggs. Identify:  $S$ ,  $F$ ,  $n$ ,  $p$ , and  $q$ .

Find and interpret the following (also, are any of the probabilities unlikely?):

b)  $P(x = 3)$

c)  $P(x \leq 3)$

d)  $P(x \geq 7)$

e)  $P(2 \leq x \leq 6)$

**Example 3**

Answer the two probabilities from the Corona virus scenario above.

**BINOMIAL PARAMETERS**

For the binomial distribution  $B(n, p)$

$$\text{Mean: } \mu = np$$

$$\text{Variance: } \sigma^2 = npq$$

$$\text{Standard deviation: } \sigma = \sqrt{npq}$$

**Example 4** Find the mean and standard deviation for the distribution of golden labs from example 1, e.g., a  $B(5, 0.35)$  distribution.