Math 146 **5.1 - Discrete Probability Distributions**

Example 1: Write down the sample space for the outcomes of flipping three coins, and create a probability distribution table and histogram for the outcomes.

DEFINITIONS

A **random variable** is a variable (typically represented by *x*) that has a single numerical value, determined by chance, for each outcome of a procedure.

A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a table, formula, or graph.

A **discrete random variable** has a collection of values that is finite or countable.

A **continuous random variable** has infinitely many values, and the collection of values is not countable.

PROBABILITY DISTRIBUTION REQUIREMENTS

1) There is a numerical random variable *x* and its values are associated with corresponding probabilities.
2) $\sum P(x) = 1$ where x assumes all possible values.
3) $0 \le P(x) \le 1$ for every value of the random variable x.

Example 2 Let *x* be the random variable for a spinner in a child's game with the following outcomes:

# of moves x	$P(x)$
0	0.20
1	0.10
2	0.25
3	0.30
4	0.15

Is this a probability distribution?

PARAMETERS OF A PROBABILITY DITRIBUTION

Since probability distributions describe the *population* the we use the mean and standard deviation parameters:

Formula 5-1 The mean: $\mu = \sum[x \cdot P(x)]$ This is also called the *expected value, i.e.,* $E(x) = \mu$.

Formula 5-2,3 The variance: $\sigma^2 = \sum\left[(x-\mu)^2 \cdot P(x)\right]$ or $\sigma^2 = \sum\left[x^2 \cdot P(x)\right] - \mu^2$

Formula 5-4 The standard deviation: $\sigma = \sqrt{\Sigma[x^2 \cdot P(x)] - \mu^2}$

(Round to one more decimal place than values used for *x*.)

Easier than the formulas, use your calculator to find the weighted mean and standard deviation with the *x* – values in **L1**, and *P*(*x*) in **L2**. Then, calculate 1-varStats **L1**, **L2**.

Computer Simulation: The following is a computer simulation of the above probability distribution using 100 spins.

In[15]:=
```
spins = Table[RandomSample[{.2, .1, .25, .3, .15} → {0, 1, 2, 3, 4}, 1], 100] // Flatten;
```

Out[∘]//TableForm=

0	3	1	4	1	2	3	3	3	3	2	0	3	3	2	3	1	3	4	3
0	0	2	1	3	3	0	2	3	3	3	2	3	0	2	0	4	4	0	0
4	0	0	3	1	3	3	1	3	3	4	4	3	4	4	3	4	0	4	0
1	1	2	2	2	2	3	2	4	0	0	3	0	2	1	2	3	3	3	3
1	3	3	1	3	3	3	2	1	0	0	0	3	0	1	1	3	2	3	4

In[16]:=
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Mean[spins] // N
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Out[16]= 2.17

In[17]:=
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StandardDeviation[spins] // N
```

Out[17]= 1.1981

Example 3: Unusual Values We can use the range rule of thumb (min = $\mu - 2\,\sigma$ and max = $\mu + 2\,\sigma$) to identify unusual values. Below is the probability distribution for x = number of tails in 10 coin tosses:

x	0	1	2	3	4	5	6	7	8	9	10
$P(x)$	0.000977	0.0098	0.0439	0.1172	0.2051	0.2461	0.2051	0.1172	0.0439	0.0098	0.000977

Parameters for this distribution: $\mu = 5$, $\sigma = 1.57$

Find the outcomes that would be considered unusual, unlikely.

A note regarding unlikely and unusual:

Example 4: Consider flipping a coin 1000 times. Is the result of of getting 502 heads unusual? Is it unlikely?

P(502 heads) = P(502 or more heads) =

P(530 heads) = P(530 or more heads) =