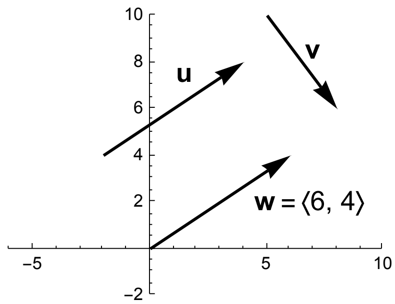
8.4a Vectors

Vector Basics

Vectors are objects that have both **magnitude** and **direction** and are typically represented by arrows in the *xy*-plane, e.g.,



A vector in **standard position** has an *initial point* at the origin and a *terminal point* indicated as $\langle a, b\rangle$. This is called **component form** of a vector. Two vectors are equal if they have the same direction and magnitude. Hence, vectors **u** and **w** above are equal vectors. Also note that vectors are denoted using bold letters.

The length (magnitude), or **modulus**, of a vector $\mathbf{v} = \langle a, b\rangle$ is

$$\|\mathbf{v}\| = \sqrt{a^2 + b^2}$$

A **unit vector** is a vector of length 1 and is computed by dividing a vector **v** by its magnitude:

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

Example 1 Find a vector in the direction of the vector $\mathbf{v} = \langle 5, -3\rangle$ that has a magnitude of 50.

The Standard Basis Vectors i and j

Two important vectors in $\mathbb{R}^2$ are $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$, and are called the *Standard Basis Vectors* for $\mathbb{R}^2$. Using the standard basis vectors, we can write vectors in algebraic form, e.g., $\mathbf{v} = 4\,\mathbf{i} + 2\,\mathbf{j} = 4\langle 1, 0\rangle + 2\langle 0, 1\rangle = \langle 4, 0\rangle + \langle 0, 2\rangle = \langle 4, 2\rangle$. This is convenient in that it allows us to manipulate vectors algebraically.

Example 2 If $\mathbf{u} = 3\,\mathbf{i} - 5\,\mathbf{j}$ and $\mathbf{v} = 6\,\mathbf{i} + 2\,\mathbf{j}$, find the vector $\mathbf{w} = 4\,\mathbf{u} + 2\,\mathbf{v}$.

Vector Addition and Subtraction, Scalar Multiplication

Vectors can be added component-wise. That is if $\mathbf{v}_1 = \langle a_1, b_1 \rangle$ and $\mathbf{v}_2 = \langle a_2, b_2 \rangle$, then $\mathbf{v}_1 + \mathbf{v}_2 = \langle a_1 + a_2, b_1 + b_2 \rangle$. A vector can also be multiplied by a scalar. Multiplying vector $\mathbf{v} = \langle a, b \rangle$ by the scalar c we have: $c\,\mathbf{v} = c\langle a, b \rangle = \langle c\,a, c\,b \rangle$.

Example 3 Given $\mathbf{u} = \langle 2, 5 \rangle$, $\mathbf{v} = \langle 7, 2 \rangle$, and $c = 3$, calculate $\boldsymbol{u} + \boldsymbol{v}$, $\mathbf{v} - \mathbf{u}$, and $c\,\mathbf{u}$. Plot all five vectors on the same axis.

Polar Form of Vectors

Often a more useful form for vectors is **polar form**. Polar form uses the magnitude and angle θ, (often called the modulus and argument) of the terminal point to write the vector $\mathbf{v} = \langle a, b \rangle$ in polar from:

$$\mathbf{v} = \langle a, b \rangle = \langle \|\mathbf{v}\| \cos(\theta), \|\mathbf{v}\| \sin(\theta) \rangle = \|\mathbf{v}\| (\cos(\theta)\,\mathbf{i} + \sin(\theta)\,\mathbf{j}), \quad \text{where} \quad \|\mathbf{v}\| = \sqrt{a^2 + b^2} \text{ and } \tan(\theta) = \frac{b}{a}.$$

Example 4 a) Find the polar form for the vector $\mathbf{v} = \langle -30, 25 \rangle$.

b) Find the component form for a force with magnitude 20 newton and direction angle 35°.

Example 5 Two force vectors, one with magnitude 120 newtons, and the second with magnitude 80 newtons, are acting on a mass and are separated by an angle of 40°. Find the magnitude and direction angle of the resultant with respect to the 120 N force. If the two forces move the mass 5 meters, find the work done by the forces on the mass. (Note: work = force × distance)