7.3 Double-Angle, Half-Angle Formulas, and Product-Sum Formulas

Objectives: (1) Use the double angle formulas to simplify expression, (2) use the half angle formula to simplify expressions.

Double-angle and half-angle are formulas involve expressions such as $\sin(2\,\theta)$, $\cos(2\,\theta)$, and $\sin\left(\frac{1}{2}\,\theta\right)$, $\cos\left(\frac{1}{2}\,\theta\right)$.

Double-Angle Formulas

$$\sin(2\,\theta) = 2\sin(\theta)\cos(\theta) \qquad (1)$$

$$\cos(2\,\theta) = \cos^2(\theta) - \sin^2(\theta) \qquad (2)$$

$$= 1 - 2\sin^2(\theta) \qquad (3)$$

$$= 2\cos^2(\theta) - 1 \qquad (4)$$

$$\tan(2\,\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)} \qquad (5)$$

Example 1 Use the sine sum formula and the cosine sum formula to derive the first two double angle formulas.

Example 2 Use the Pythagorean Identity to derive the last two double angle formulas.

Example 3 Suppose $\sin(x) = \frac{2}{7}$ and $\frac{\pi}{2} \le x \le \pi$. Find $\sin(2\,x)$.

Example 4 Rewrite $y = \sin(3\,x)\cos(3\,x)$ as a single sine function.

Example 5 Write $\cos(3x)$ in terms of $\cos(x)$.

Power Reducing Formulas

Example 6 Rearrange formulas 3 and 4 to derive formulas for $\sin^2(x)$, $\cos^2(x)$, and $\tan^2(x)$. These are called **power reducing formulas**.

Example 7 Write $\cos^2(x)\sin^2(x)$ in terms of cosine without powers.

Half Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} = \frac{1-\cos(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{1+\cos(\theta)}$$

The choice of + or - depends on the quadrant $\frac{\theta}{2}$ is in .

Example 7 Find the exact value of $\cos(22.5°)$.