

7.2 Addition and Subtraction Formulas

Objectives: To derive and use the sum and difference formulas for: (1) $\cos(\alpha \pm \beta)$, (2) $\sin(\alpha \pm \beta)$, and (3) $\tan(\alpha \pm \beta)$.

Often we have the situation where we need to simplify a trig function whose argument is the sum or difference of two angles, e.g., $\cos(\alpha + \beta)$ or $\sin(\alpha - \beta)$, etc.

The Cosine Difference Formula

The **Cosine Difference** formula states

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

Proof:

Example 1 Use the cosine difference formula to find the exact value of $\cos(15^\circ)$.

The Cosine Sum Formula

The **Cosine Sum** formula states

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Example 2 Derive the *cosine sum* formula by substituting $-\beta$ for each β in the cosine difference formula.

Example 3 Find the exact value of $\cos(105^\circ)$.

Sine Sum and Difference Formulas

Example 4 Substitute $\frac{\pi}{2} - \beta$ for β and $-\alpha$ for α in the cosine sum formula to derive the **sine sum** formula.

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

Example 5 Substitute $-\beta$ for each β in the sine difference formula to derive the **sine difference** formula

$$\sin(\alpha - \beta) = \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$$

Example 6 Simplify $\sin(x + \frac{\pi}{2})$.

Sum and Difference Formulas for Tangent

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha) \tan(\beta)}$$