

## 7.1 Trigonometric Identities

**Objectives:** (1) Use algebra to simplify trigonometric expressions, (2) prove trigonometric identities.

### Identities

An equation  $f(x) = g(x)$  which is true **for all  $x$ 's** is called an identity. An equation that is not true for every value of  $x$  is called a *conditional equation*.

We have already developed several basic trigonometric identities:

The identities can be used in various forms. For instance, you may need to use  $\sec^2(x) - 1$  to replace  $\tan^2(x)$ . Knowing the identities is more useful than just memorizing them.

**Example 1** Establish (or prove) the identity:  $\frac{\sin^2(x)}{1-\cos(x)} - 1 = \cos(x)$

**Example 2** Prove the identity:  $\csc(x) - \cot(x) = \frac{\sin(x)}{1+\cos(x)}$

**Example 3**

Prove the identity:  $\frac{\sin^3(x) + \cos^3(x)}{\sin(x) + \cos(x)} = 1 - \sin(x) \cos(x)$

**Example 4**

Prove the identity:  $\frac{1}{1 - \sin(x)} - \frac{1}{1 + \sin(x)} = 2 \sec(x) \tan(x)$

**Example 5**

Prove the identity:  $\frac{\sin(x) + \cos(x)}{\sin(x)} - \frac{\cos(x) - \sin(x)}{\cos(x)} = \sec(x) \csc(x)$

**Example 6**

Use the substitution  $x = 2 \tan(\theta)$  to simplify  $\frac{1}{x^2 \sqrt{4+x^2}}$

## Cofunctions

In trigonometry **cofunctions** are two functions,  $f$  and  $g$ , that are equal when evaluated with complimentary angles,  $\alpha$  and  $\beta$ , (i.e.,  $\alpha + \beta = 90^\circ = \frac{\pi}{2}$  rad), that is:  $f(\alpha) = g(\beta)$ .

**Example 7**

Show that sine and cosine are cofunctions. Find the other pairs of cofunctions.