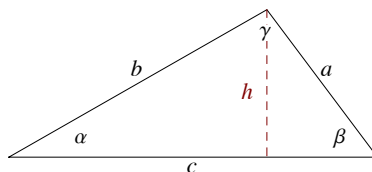


6.5 The Law of Cosines

Objectives: (1) Solve SAS problems, (2) Solve SSS problems, (3) Solve applied problems.

The Law of Cosines

We saw that The Law of Sines is used to solve a triangle problem whenever an angle-side pair is given. If no angle-side pair is given we need to use **The Law of Cosines**. Given the triangle



the **Law of Cosines** states

$$c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$

$$b^2 = a^2 + c^2 - 2ac \cos(\beta)$$

$$a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

Proof of Law of Cosines

If a triangle is placed on the rectangular coordinate grid with one corner at the origin and a second corner on the positive x-axis, the third corner can be expressed with sines and cosine, even when the corner is in the 2nd quadrant.

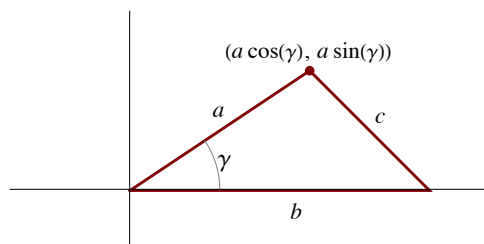


Figure 1

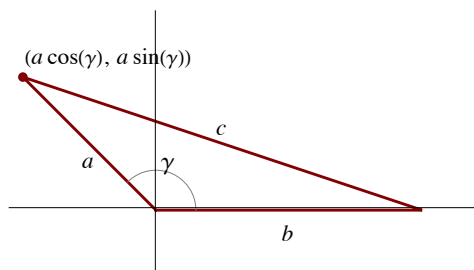
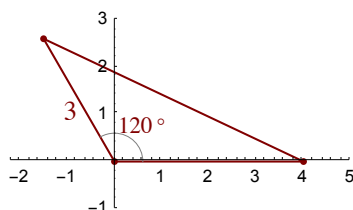


Figure 2

Example 1 Find the coordinates of each corner point for the given triangle:



Proof of Law of Cosines

Example 2 Solve the triangle with $b = 10$, $c = 15$, and $\alpha = 42^\circ$.

Example 3 Find the angles of the triangle with sides $a = 10$, $b = 15$, and $c = 20$. Also find the area of the triangle.

Example 4 A ship sails from a port with a heading due north for 42 miles then turns with a bearing $N 33^\circ E$ for 25 miles. Find the bearing and the distance the ship requires to return to port.

Example 5 The distance from the pitching mound to home plate on a major league baseball diamond is 60 feet 6 inches. The distance from home plate to first base is 90 feet. Find the distance from the pitching mound to first base. Assume the bases make a diamond (i.e., a rotated square.)