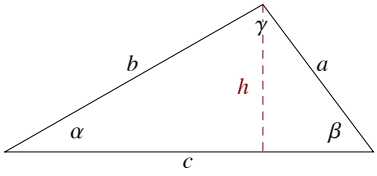
6.4 The Law of Sines

Objectives: (1) Solve ASA and SAA problems, (2) Solve SSA problems, (3) Solve applied problems.

Consider the following triangle with sides *a*, *b*, and *c*, and corresponding angles α, β, and γ :



Using right triangle trig we can show $\sin(\alpha) = \frac{h}{b}$ and $\sin(\beta) = \frac{h}{a}$. Therefore, $b\sin(\alpha) = a\sin(\beta)$, or that $\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b}$.

The Law of Sines

For the triangle given above, ***The Law of Sines*** state

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

The Law of Sines can be used to solve a triangle whenever an angle and its corresponding side are known.

Example 1 Solve the triangle with $c = 15$, $\beta = 38°$, and $\gamma = 102°$. (This is known as an SAA problem.)

Example 2 A triangle has two angles of 24° and 32°, with an included side of 7.5. Solve the ASA triangle.

Solving an SSA Problem

- Solving an SSA triangle (two sides and the angle **is not** an interior angle) gets a little more complicated in that there may be (1) one solution, (2) two solutions, or (3) no solutions.

Example 3 Solve the triangle given $a = 14$, $c = 11$, and $\alpha = 40°$.

Example 4 Solve the triangle given $a = 12$, $c = 17$, and $\alpha = 30°$.

Example 5 Solve the triangle given $a = 14$, $c = 20$, and $\alpha = 45°$.