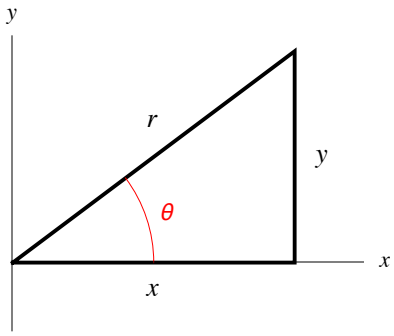
6.3 Trigonometric Functions of Angles

Definition of the Trigonometric Functions

Example 1 Use the following triangle to write the six trigonometric functions in terms of θ, x, y, and r.



Example 2 Suppose $\tan(\theta) = -\frac{2}{5}$ and θ is in quadrant II. Find

a) $\cos(\theta)$ b) $\csc(\theta)$ c) θ in degrees

Fundamental Identities

I. Reciprocal Identities

$\csc(\theta) = \frac{1}{\sin(\theta)}$ $\sec(\theta) = \frac{1}{\cos(\theta)}$ $\cot(\theta) = \frac{1}{\tan(\theta)}$ $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

II. Pythagorean Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$ $\tan^2(\theta) + 1 = \sec^2(\theta)$ $\cot^2(\theta) + 1 = \csc^2(\theta)$

Example 3 Use the Pythagorean theorem on the triangle in example 1 to prove the Pythagorean Identity.

III. Even-Odd Identities

$\sin(-\theta) = -\sin(\theta)$ $\cos(-\theta) = \cos(\theta)$ $\tan(-\theta) = -\tan(\theta)$

Area of a Triangle

From basic geometry, the area of a triangle is $A = \frac{1}{2}\text{base} \times \text{height}$. We can also find the area of a triangle not knowing the height as long as we know two sides and the included angle.

Example 4 Find the expression for the area of a triangle with sides a, b, and included angle θ.

Example 5 Find the area of the triangle with base $c = 5$, $a = 8$, and $\beta = 20°$.

Example 6 Find the area shaded region below called a *lens*. Find a formula for a lens for any r and θ (in radians).