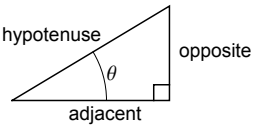
6.2 Trigonometry of Right Triangles

Objectives: (1) Convert from degrees to radians, (2) Define the six trigonometric functions, (3) Solve right triangle problems.

Trigonometric Functions: Sine, Cosine, Tangent, Cosecant, Secant, Cotangent

The word *trigonometry* is derived from the Greek words trigōnon and metron, meaning "triangle measure", or the study of triangles and the relationships of their sides and angles. Since a triangle is made up of three sides, a total of six ratios using two sides at a time can be made. Consider the right-triangle below:



the six ratios, or trigonometric functions relative to the angle θ, are defined as:

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}} \qquad \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} \qquad \sec(\theta) = \frac{\text{hyp}}{\text{adj}} \qquad \cot(\theta) = \frac{\text{adj}}{\text{opp}}$$

Example 1 A right triangle has base 5, height 3, and angle θ opposite 3. Find the hypotenuse, and the six trigonometric functions of θ.

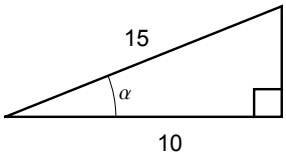
Example 2 A right triangle has a height of 17.5 cm, and an opposite angle of 38°. Find the remaining angles and sides of the triangle.

Example 3 A UFO is spotted at an angle of elevation of 47° from point *A*. An second observer, at point *B*, is 800 feet directly behind observer *A* and has an angle of elevation of 39°. Find the height of the UFO above the ground.

Inverse Functions of Sine, Cosine, and Tangent

For a right triangle, an unknown angle can be found using *inverse trig functions*, if two sides of the triangle are known. The inverse trigonometric functions for sine, cosine and tangent are *arcsine*, *arccosine*, and *arctangent* (more on these in other chapters.)

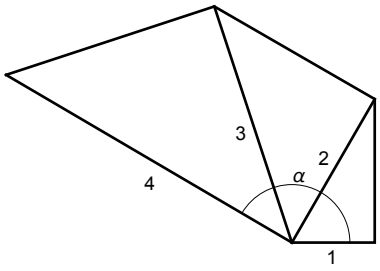
For example: to find the included angle for a right triangle with hypotenuse 15 base 10, we first draw a picture:

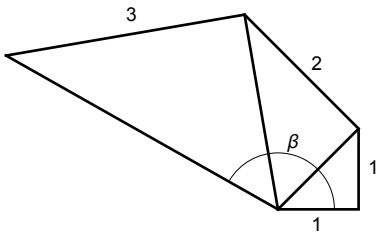


and notice that the adjacent and hypotenuse is given, meaning $\cos(\alpha) = \frac{10}{15}$, or that $\alpha = \cos^{-1}\left(\frac{10}{15}\right) \Rightarrow \alpha \approx 48.19°$.

Example 4 Find the angle of elevation for a 10 foot ladder leaning against a wall if the top of the ladder has a height 9 feet 6 inches. Also, find the distance the base of the ladder is from the wall.

Example 5 Find the angles α and β in each figure below. Note: all the triangles are right triangles.





Example 6 A woman is standing on a hill and sees a building that she knows is 60 feet tall. The angle of depression to the bottom of the building is 14°, and the angle of elevation to the top of the building is 18°. Find her distance from the building.