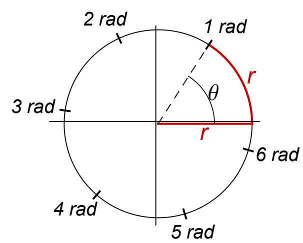
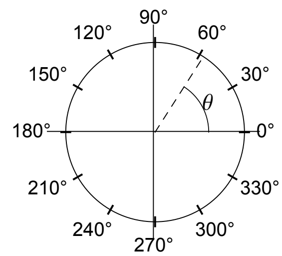
6.1 Angle Measure; Area of Sectors; Linear and Angular Speed

Objectives: (1) Convert between radians and degrees, (2) calculate arc length, (3) calculate the area of a sector, (4) calculate linear and angular speed.

Measuring Angles: Degrees -vs- Radians

- Most people know the measure of angles in degrees, whereby there are 90° in a *right angle*, 180° in a half circle, and 360° in a full circle. Another method of measuring an angle is using **radians**. Consider a circle with radius *r*. Since the circumference of the circle is $C = 2\pi r$, we can mark the perimeter of the circle in increments of the radius *r*, (or radian)





Converting Between Degrees and Radians

Since one full circle is 2π rad or 360°, a half a circle gives $\pi \text{ rad} = 180°$, or the conversion factors: $\frac{\pi \text{ rad}}{180°} = \frac{180°}{\pi \text{ rad}}$; or $1° = \frac{\pi}{180}$ rad, and $1 \text{ rad} = \frac{180}{\pi}°$.

Example 1 a) Convert 1 rad into degrees, b) $\frac{\pi}{3}$ rad to degrees c) 132° into radians.

Arc Length

Recall that the circumference of a circle is $C = 2\pi r$. If, instead, we don't have a full circle (i.e., a full 2π radians), but instead θ radians, there would be only $\frac{\theta}{2\pi}$ of the circumference, or a length of $\frac{\theta}{2\pi} 2\pi r = \theta r$.

For an arc with radius *r*, and an angle of θ radians, the **arc length** *s* is given by

$$s = r\theta$$

Example 2 A road has a circular corner with a radius of 120 feet and travels through an angle of 72°. Find the length of the circular part of the corner.

Example 3 Mount Vernon and San Francisco have roughly the same longitude. The latitude of Mount Vernon is 48°25'17'', and the latitude of San Francisco is 37°46'30''. Given the diameter of the earth is 7918 miles, find the distance between Mount Vernon and San Francisco.

Area of a Sector

Just as the arc length can be calculated as a portion of a full circle, so can the area of a sector of a circle. The area of a sector of a circle of radius r and angle θ is

$$A = \frac{1}{2}\theta r^2$$

Example 4 Which is larger, a piece of pizza from one with a diameter of 16 inches and an angle of 45°, or from a 20 inch diameter pizza and an angle of 30°?

Linear and Angular Speed

Recall that the linear speed of an object is given by $v = \frac{\text{distance}}{\text{time}} = \frac{s}{t}$, i.e., $\text{mph} = \frac{\text{miles}}{\text{hour}}$. For an object that is rotating or spinning, we define the *angular* speed as the number of rotations per time (i.e., rpm, rotations per minute) or more precisely, the number of radians per unit of time:

$$\omega = \frac{\theta}{t}$$

This means that the *linear* speed of a particle with radius r and angular speed ω is: $v = \frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = r\omega$:

$$v = r\omega$$

Linear and Angular Speed Manipulate

Example 5 A toy car travels around a record at a rate of 33 1/3 RPM.

a) Find its angular speed
b) Find its linear speed when it is 5.5 inches from the center.

Example 6 A bicycle has a larger front sprocket with radius 4.5 inches and a smaller rear sprocket radius of 1.75 inches. The tires have a diameter of 26 inches.

a) If a cyclist pedals 50 RPM's, find the angular speed of the tires and the linear speed of the bicycle.
b) How fast does the cyclist need to pedal to reach 35 miles per hour?