5.5 Modeling Harmonic Motion

Objectives: (1) Build a model for simple harmonic motion, (2) Analyze simple harmonic motion, (3) Analyze damped harmonic motion, (4) Graph the sum of two functions.

Simple Harmonic Motion

In mechanics and physics, Simple Harmonic Motion is a type of periodic motion where the restoring force is directly proportional to the displacement. *Examples*: a mass on a spring, a pendulum, the vibration of a molecule, the vibration of a guitar string.

Consider a mass of m suspended on a spring with spring constant k. We can model the position of the mass by the formula

$$s(t) = a\cos(\omega t)$$

The **amplitude**, a, of the simple harmonic motion is

$$\text{amplitude} = |\, a \,|$$

The **frequency**, f, of the simple harmonic motion is

$$f = \frac{\omega}{2\pi} \qquad \omega > 0$$

If the **spring constant**, k, and the **mass**, m, is known, then we also have

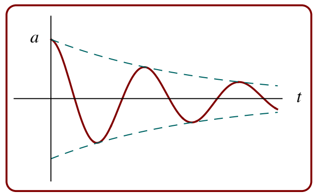
$$\omega = \sqrt{\frac{k}{m}}$$

Example 1 A spring is suspending a 3.2 kg mass and is raised 35 cm above equilibrium. After the mass is released, it takes 1.18 seconds to return to its starting position. Find a model for the height of the mass after time t, and find the spring constant, k.

Example 2 A 100 g mass is attached to a spring with a spring contstant 0.8 N/m and is pulled down 80 cm from its equilibrium position. Find an equation that models the position of the mass at anytime t. Find its position and direction of movement 18 second after being released.

Damped Harmonic Motion

Most physical systems are affected by friction or some other resistive force. Typically, a periodic motion has an amplitude that decreases with time due to the resistance.



Not only is the amplitude affected by the resistance, but also the period. The model the gives the displacement of a mass m in damped-harmonic motion with a **damping coefficient** b is given by

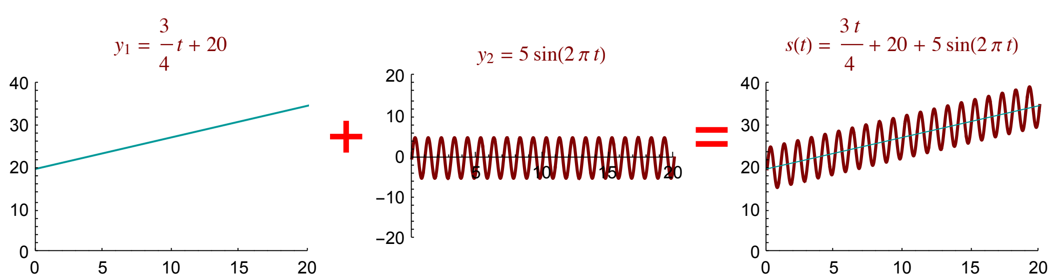
$$s(t) = a\,e^{-b\,t/(2\,m)}\cos\left(\sqrt{\omega^2 - \frac{b^2}{4\,m^2}}\;t\right)$$

Note that if $b = 0$, we get the simple harmonic motion model $s(t) = a\cos(\omega\,t)$.

Example 3 Suppose a pendulum with mass 0.4 kg without resistance has an amplitude of 30 cm and a period of 4.5 seconds. If the pendulum has a damping coefficient of 15 N/m and is pulled to the left (negative displacement), find the model that gives its position after time t, and find its position and direction after 30 seconds.

Combining Functions

Often, simple harmonic motion can be combined into a more generalized periodic motion by combining functions. Consider modeling the temperature of a town that has an average increase in temperature from 20° to 35° over a period of 20 days, but also has a daily fluctuation of $\pm 5°$. Assuming the starting temperature is 20° and is rising, we can combine the linear rise in average temperature with the periodic fluctuating daily temperature by addition:



Example 4 Suppose that electricity usage for an average household is about 1.4 kw per hour on the weekend and decreases to about 1 kw per hour mid week. Assume the usage is periodic. Also, suppose daily electricity fluctuates with the greatest usage around 6AM and 6PM, and the fluctuation is 0.4 kw per hour (i.e., ± 0.2 kw per hour). Find a model for electricity usage and graph the function for a two-week period.