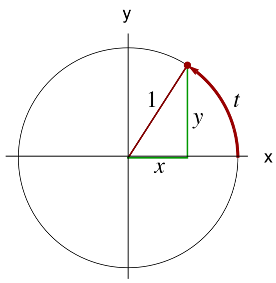
# 5.2 Trigonometric Functions of Real Numbers

**Objectives**: (1) Use the unit circle to define the trig functions, (2) use the unit circle to find exact values of trig functions.

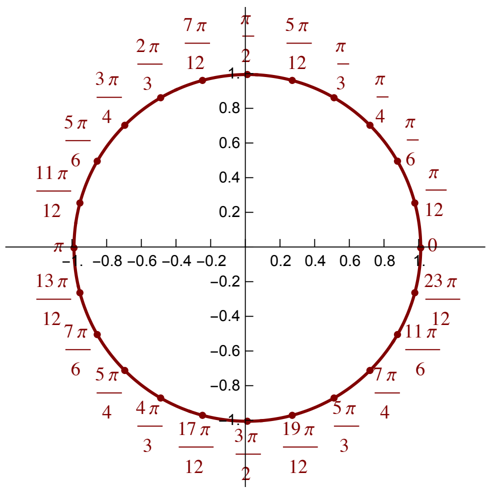
If a right triangle is placed inside the unit circle, we can label the sides of the triangle with the coordinates of the terminal point for any number *t*.

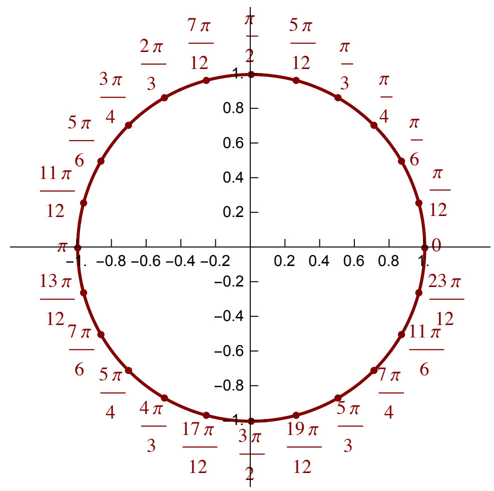



$$\sin(t) = y \quad \cos(t) = x \quad \tan(t) = \frac{y}{x}$$

$$\csc(t) = \frac{1}{y} \quad \sec(t) = \frac{1}{x} \quad \cot(t) = \frac{x}{y}$$

**Example 1** Find the six trigonometric functions of the real number *t* in the table:





| $t$ | $\sin(t)$ | $\cos(t)$ | $\tan(t)$ | $\csc(t)$ | $\sec(t)$ | $\cot(t)$ |
|---|---|---|---|---|---|---|
| $0$ | | | | | | |
| $\frac{\pi}{6}$ | | | | | | |
| $\frac{\pi}{4}$ | | | | | | |
| $\frac{\pi}{3}$ | | | | | | |
| $\frac{\pi}{2}$ | | | | | | |

## The Domain and Range of Trigonometric Functions

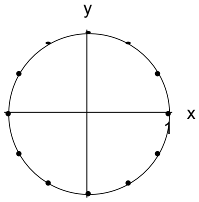
**Example 2** Determine the domain of the six trigonometric functions:

(a) sin(*t*) (b) cos(*t*) (c) tan(*t*) (d) csc(*t*) (e) sec(*t*) (f) cot(*t*)

## Odd and Even Functions

Recall the definition of an **even** function is $f(-x) = f(x)$, and the definition of an **odd** function is $f(-x) = -f(x)$.

**Example 3** Use the unit circle to determine whether sin($t$), cos($t$), and tan($t$), are either odd, even, or neither. Note: this will also determine whether csc($t$), sec($t$), and cot($t$) are odd or even.




## Fundamental Identities

Using the unit circle definitions of sine and cosine, rewrite tan($t$) in terms of sine and cosine:

$$\tan(t) = \_\_\_\_\_\_\_\_\_\_$$

**Example 4** Write csc($t$), sec($t$), and cot($t$) in terms of sine and cosine.

Recall the equation for a circle with radius 1 is $x^2 + y^2 = 1$. Using the definition for cosine and sine, we have

$$\cos^2(t) + \sin^2(t) = 1$$

This identity is call *The Pythagorean Identity*.

**Example 5** Use the Pythagorean Identity to derive two more Pythagorean Identities by dividing by $\cos^2(x)$, and by $\sin^2(x)$.

**Example 6** Suppose $\sec(t) = \frac{-7}{3}$ where $t$ is in quadrant II. Find the value of the five other trig functions of $t$.