

10.5 Rotation of Axes

Recall the general form of a conic is given by

$$A x^2 + C y^2 + D x + E y + F = 0$$

Missing from that expression is the term $B x y$.

$$A x^2 + B x y + C y^2 + D x + E y + F = 0$$

The Angle of Rotation

The affect of the xy term is a rotation (and scaling) of the conic, and can also effect the shape. The rotation angle of the conic is found by solving for the acute angle θ in the equation:

$$\cot(2\theta) = \frac{A-C}{B}$$

Rotated Coordinate System

Using a new rotated coordinate system (X, Y) , we can remove the rotation xy -term. Equations to convert from (x, y) to (X, Y) are:

$$\begin{array}{l} x = X \cos(\theta) - Y \sin(\theta) \\ y = X \sin(\theta) + Y \cos(\theta) \end{array} \quad \text{and} \quad \begin{array}{l} X = x \cos(\theta) + y \sin(\theta) \\ Y = -x \sin(\theta) + y \cos(\theta) \end{array}$$

The Discriminant

The xy term in the general conic equation also affects the type of conic. To identify the type conic we use the discriminant $B^2 - 4AC$.

1. If $B^2 - 4AC = 0$ the graph is a **parabola**.
2. If $B^2 - 4AC < 0$ the graph is an **ellipse**.
3. If $B^2 - 4AC > 0$ the graph is a **hyperbola**.

Example 1 Identify the conic and angle of rotation for: $4x^2 + 6xy + 5y^2 - 8x + 7y - 20 = 0$.

Example 2 Solve the equation in *Example 1* for y and graph the conic on your calculator along with the rotated axes.

Example 3 Identify and sketch the curve $73x^2 + 72xy + 52y^2 + 30x - 40y - 75 = 0$. To find exact values for $\sin(\theta)$ and $\cos(\theta)$ half-angle formulas are useful. Find the XY -coordinates of the foci, and then find the xy -coordinates of the foci.