

10.4 Shifted Conics

Recall that the transformation $y = f(x - h)$ translates a function h units to the right, and $y = f(x) + k$ translates the function up k units. If we subtract k from each side of the second transformation we can write vertical transformation as $y - k$. This is the reason why a circle with radius r centered at the origin has equation

$$x^2 + y^2 = r^2$$

and a circle with radius r with center at (h, k) has the equation

$$(x - h)^2 + (y - k)^2 = r^2$$

Similar transformations can be done with all the conic sections.

Shifted Conics

Definition: The set of all points equidistant from a fixed point.

$$\begin{array}{ll} x^2 + y^2 = r^2 & \text{radius} = r, \text{ center} = (0, 0) \\ (x - h)^2 + (y - k)^2 = r^2 & \text{radius} = r, \text{ center} = (h, k) \end{array}$$

The Parabola

Definition: The set off all points equidistant from a fixed point and a fixed line.

vertical:	$x^2 = 4py$	vertex = (0, 0),	focus = (0, p),	directrix: $y = -p$
	$(x - h)^2 = 4p(y - k)$	vertex = (h, k),	focus = (h, k + p)	directrix: $y = k - p$
horizontal:	$y^2 = 4px$	vertex = (0, 0),	focus = (p, 0),	directrix: $x = -p$
	$(y - k)^2 = 4p(x - h)$	vertex = (h, k),	focus = (p + h, k)	directrix: $x = h - p$

The Ellipse

Definition: The set of all points whose sum of distances from two fixed points is constant.

$$\begin{array}{ll} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 & \text{center} = (0, 0) \quad \text{if } a > b \text{ major axis int: } x = (\pm a, 0) \quad \text{minor axis int: } y = (0, \pm b) \\ & \text{eccentricity: } e = \frac{c}{a} \text{ (or } e = \frac{c}{b} \text{ if } b > a) \quad a^2 - b^2 = c^2 \text{ if } a > b, \text{ or } b^2 - a^2 = c^2 \text{ if } b > a \\ \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 & \text{center} = (h, k) \end{array}$$

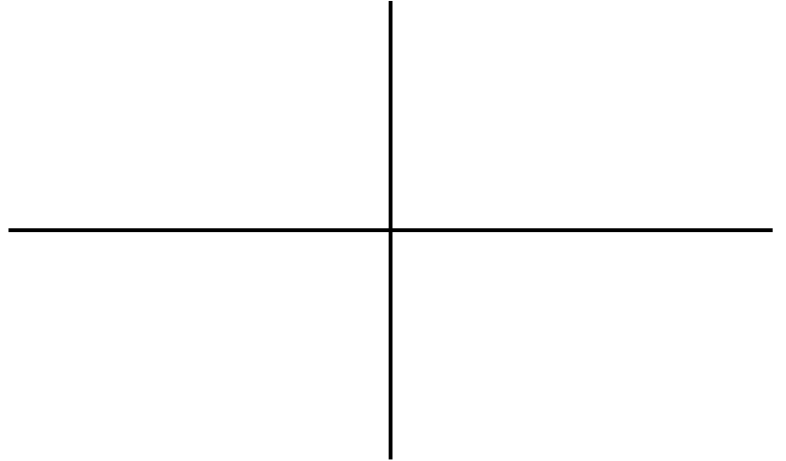
The Hyperbola

Definition: The set of all points whose difference of distances from two fixed points is constant.

horizontal:	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	center = (0, 0)	major axis int: $x = (\pm a, 0)$	eccentricity: $e = \frac{c}{a}$	$a^2 + b^2 = c^2$
	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	center = (h, k)			
vertical:	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$	center = (0, 0)	major axis int: $x = (\pm b, 0)$	eccentricity: $e = \frac{c}{b}$	$a^2 + b^2 = c^2$
	$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$	center = (h, k)			

Example 1

Graph the conic given by $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1$. Label all important features (center, vertices, foci, asymptotes, etc.)



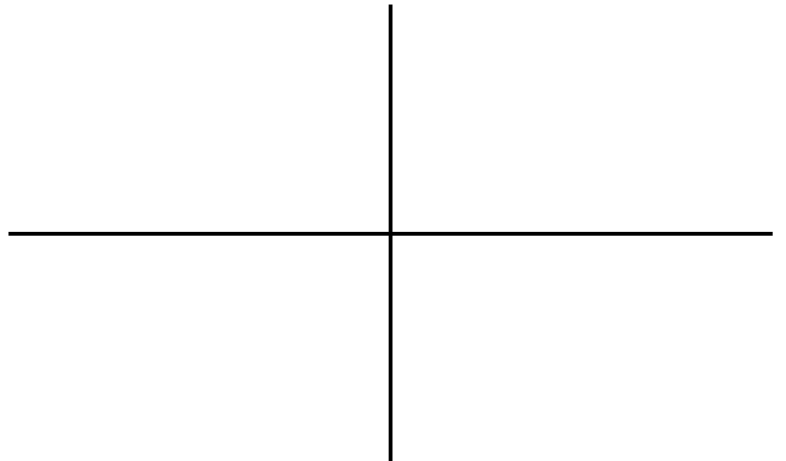
All of the equations in standard form can be expanded to the general form:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Completing the square allows us to rewrite the conic into standard form.

Example 2

Graph the conic given by $20x^2 - 9y^2 + 160x + 54y + 59 = 0$. Label all important features (center, vertices, foci, asymptotes, etc.)

**Example 3**

Graph the conic given by $25x^2 + 4y^2 + 250x - 16y + 541 = 0$. Label all important features (center, vertices, foci, asymptotes, etc.)

