10.4 Shifted Conics

Recall that the transformation $y = f(x - h)$ translates a function h units to the right, and $y = f(x) + k$ translates the function up k units. If we subtract k from each side of the second transformation we can write vertical transformation as $y - k$. This is the reason why a circle with radius r centered at the origin has equation

$$x^2 + y^2 = r^2$$

and a circle with radius r with center at (h, k) has the equation

$$(x - h)^2 + (y - k)^2 = r^2$$

Similar transformations can be done with all the conic sections.

Shifted Conics

Definition: **The set of all points equidistant from a fixed point.**

$x^2 + y^2 = r^2$	radius $= r$, center $= (0, 0)$
$(x - h)^2 + (y - k)^2 = r^2$	radius $= r$, center $= (h, k)$

The Parabola

Definition: The set off all points equidistant from a fixed point and a fixed line.

vertical:	$x^2 = 4py$	vertex $= (0, 0)$, focus $= (0, p)$,	directrix : $y = -p$
	$(x - h)^2 = 4p(y - k)$	vertex $= (h, k)$, focus $= (h, k + p)$	directrix: $y = k - p$
horizontal:	$y^2 = 4px$	vertex $= (0, 0)$, focus $= (p, 0)$,	directrix: $x = -p$
	$(y - k)^2 = 4p(x - h)$	vertex $= (h, \text{k0})$, focus $= (p + h, k)$	directrix: $x = h - p$

The Ellipse

Definition: The set of all points whose *sum* of distances from two fixed points is constant.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ center $= (0, 0)$ if $a > b$ major axis int: $x = (\pm a, 0)$ minor axis int: $y = (0, \pm b)$

eccentricity: $e = \frac{c}{a}$ (or $e = \frac{c}{b}$ if $b > a$) $a^2 - b^2 = c^2$ if $a > b$, or $b^2 - a^2 = c^2$ if $b > a$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ center $= (h, k)$

The Hyperbola

Definition: The set of all points whose *difference* of distances from two fixed points is constant.

horizontal: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ center $= (0, 0)$ major axis int: $x = (\pm a, 0)$ eccentricity: $e = \frac{c}{a}$ $a^2 + b^2 = c^2$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ center $= (h, k)$

vertical: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ center $= (0, 0)$ major axis int: $x = (\pm b, 0)$ eccentricity: $e = \frac{c}{b}$ $a^2 + b^2 = c^2$

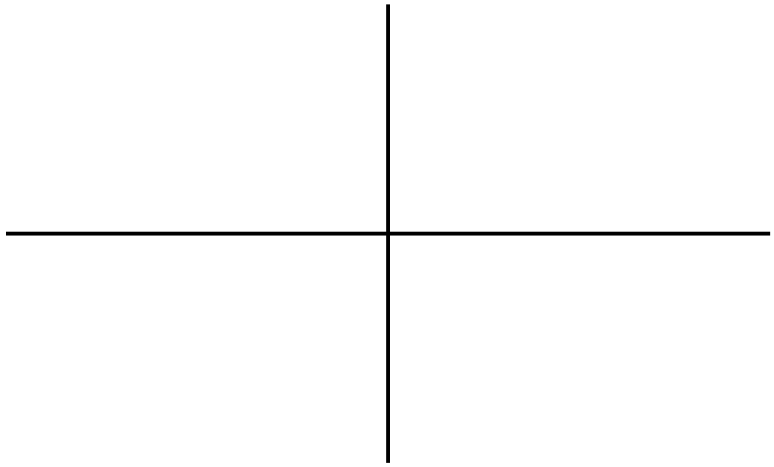
$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$ center $= (h, k)$

All of the equations in standard form can be expanded to the general form:

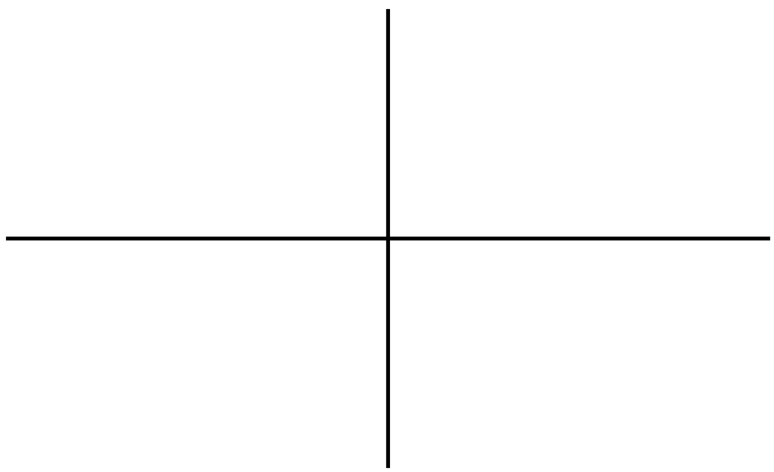
$$A\,x^2 + C\,y^2 + D\,x + E\,y + F = 0$$

Completing the square allows us to rewrite the conic into standard form.

Example 1 Graph the conic given by $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1$. Label all important features (center, vertices, foci, asymptotes, etc.)



Example 2 Graph the conic given by $20\,x^2 - 9\,y^2 + 160\,x + 54\,y + 59 = 0$. Label all important features (center, vertices, foci, asymptotes, etc.)



Example 3 Graph the conic given by $25\,x^2 + 4\,y^2 + 250\,x - 16\,y + 541 = 0$ Label all important features (center, vertices, foci, asymptotes, etc.)