

1.6 Modeling with Equations

Example 1 John and Susan make \$84925 per year combined. If Susan makes 15% more than John, find each of their yearly salaries.

Let $x = \text{John's salary}$
 $1.15x = \text{Susan's salary}$

$$\begin{aligned}x + 1.15x &= 84925 \\2.15x &= 84925 \\x &= \frac{84925}{2.15}\end{aligned}$$

John makes \$39500/yr
 Susan makes \$45425/yr.

$$\leftarrow = 39,500$$

Example 2 The Law of the Lever states that two masses are balanced using a lever and fulcrum when the product of each mass by its distance from the fulcrum are equal, that is, $m_1 d_1 = m_2 d_2$. Suppose a 120 pound mom and her 50 pound son are playing on a 15 foot long seesaw. Find the distance each needs to be from the fulcrum so that the seesaw is balanced. (Round to one decimal place.)



$$\begin{aligned}120x &= 50(15-x) \\120x &= 750 - 50x \\170x &= 750 \\x &= \frac{750}{170} \\x &= 4.4 \text{ ft}\end{aligned}$$

Mom needs to be 4.4 ft and son needs to be 10.6 ft from the fulcrum.

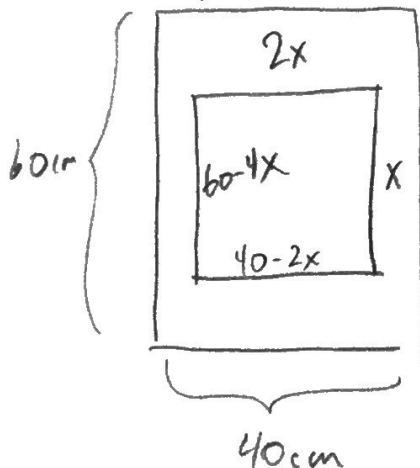
Example 3 Find three consecutive integers such that the product of the first with the sum of the last two is 324.

1st int = x
 2nd int = $x+1$
 3rd int = $x+2$

$$\begin{aligned}x(x+1+x+2) &= 324 \\x(2x+3) &= 324 \\2x^2 + 3x - 324 &= 0 \\(2x+27)(x-12) & \\x = \frac{-27}{2} \quad x = 12\end{aligned}$$

Integers are $\{12, 13, 14\}$

Example 4 A vertical sheet of paper measures 40 cm by 60 cm. The margin for the printed area at the top and bottom needs to be twice the measurement as the side margins. Find the width of the margins so that the printed area is exactly half the area of the entire sheet of paper. (What if the sheet was horizontal?)



$$\begin{aligned}(60-4x)(40-2x) &= \frac{1}{2}(60)(40) \\8x^2 - 280x + 2400 &= 1200 \\8x^2 - 280x + 1200 &= 0 \\x^2 - 35x + 150 &= 0 \\(x-30)(x-5) &= 0 \\x = 30 \quad \text{or} \quad x = 5\end{aligned}$$

Top border is 10cm, side border is 5cm!

Example 5 The surface area of a sphere is $A = 4\pi r^2$ and the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Write the volume in terms of surface area, and simplify.

$$A = 4\pi r^2$$

$$\sqrt{\frac{A}{4\pi}} = r$$

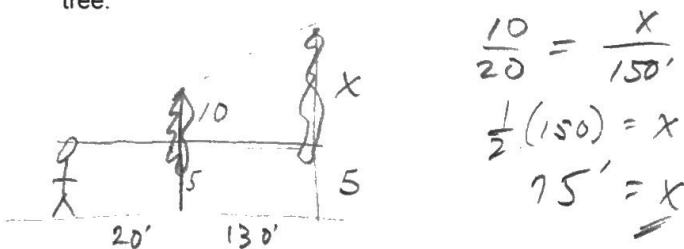
$$V = \frac{4}{3}\pi \left(\sqrt{\frac{A}{4\pi}}\right)^3$$

$$V = \frac{4}{3}\pi \frac{A^{3/2}}{4^{3/2}\pi^{3/2}}$$

$$V = \frac{4}{3 \cdot 8 \pi^{1/2}} A^{3/2}$$

$$V = \frac{A^{3/2}}{6\pi^{1/2}}$$

Example 6 Two trees are 130 feet apart. When Hailee stands 20 feet from the smaller tree the tip of the tree lines up with the taller tree. Assuming Hailee's eye level is 5 feet and the smaller tree is 15 feet tall, find the height of the taller tree.



$$\frac{10}{20} = \frac{x}{150}$$

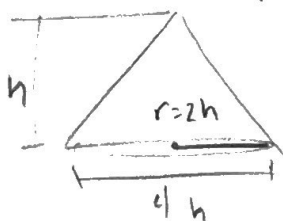
$$\frac{1}{2}(150) = x$$

$$75' = x$$

The taller tree is 80 feet tall.

similar triangles

Example 7 A pile of topsoil makes a cone whose diameter is 4 times the height. If the pile contains 15 cubic yards of soil, how tall is the pile in feet.



$$V = \frac{1}{3}\pi r^2 h$$

$$15 = \frac{1}{3}\pi (2h)^2 h$$

$$45 = \pi 4 h^3$$

$$\frac{45}{4\pi} = h^3$$

$$h = \sqrt[3]{\frac{45}{4\pi}}$$

$$= 1.53 \text{ yds}$$

$h = 4.59$ feet tall

Example 8 A 200 inch long wire is cut into two pieces. One piece is bent into a circle, and the other is bent into a square. How much wire should be in each piece so that the two shapes have the same area? Round to two decimal places.



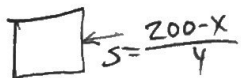
$$C = 2\pi r$$

$$x = 2\pi r$$

$$\frac{x}{2\pi} = r$$

$$A = \pi r^2$$

$$A = \pi \left(\frac{x}{2\pi}\right)^2$$



$$A = s^2$$

$$A = \left(\frac{200-x}{4}\right)^2$$

$$\pi \left(\frac{x}{2\pi}\right)^2 = \left(\frac{200-x}{4}\right)^2$$

$$4\pi \cdot \frac{x}{2\pi} = \frac{200-x}{4} \cdot 4\pi$$

$$2\sqrt{\pi} x = 200\pi - x\pi$$

$$2\sqrt{\pi} x + \pi x = 200\pi$$

$$x(2\sqrt{\pi} + \pi) = 200\pi$$

$$x = \frac{200\pi}{2\sqrt{\pi} + \pi}$$

$$x \approx 93.97 \text{ feet}$$

93.97 in in circle
106.03 in in square