# 1.5 Solving Equations

**Objectives**: Solve linear equations, radical equations, rational equations, quadratic equations.

## Linear Equations

**Example 1** Solve the linear equation: $\sqrt{3}\,x + \sqrt{12} = \frac{x+3}{\sqrt{3}}$

$\sqrt{3}\left(\sqrt{3}x + \sqrt{12}\right) = \sqrt{3}\,\frac{x+3}{\sqrt{3}}$

$3x + 6 = x + 3$

$2x = -3$

$x = -3/2$

**Example 2** Solve the quadratic equation by completing the square: $x^2 - 8x - 5 = 0$

$x^2 - 8x + 16 = 5 + 16$

$(x-4)^2 = 21$

$x - 4 = \pm\sqrt{21}$

$x = 4 \pm \sqrt{21}$

### The Quadratic Formula

The roots of the quadratic equation $a x^2 + b x + c = 0$ where $a \neq 0$, is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **discriminant** is $b^2 - 4ac$ and determines the nature of the roots. Let $D = b^2 - 4ac$.

1. If $D > 0$ there are two real roots.
2. If $D = 0$ there is one real root.
3. If $D < 0$ there are two complex roots.

**Example 3** Solve the quadratic equation: $3x^2 + 6x - 2 = 0$.

$x = \frac{-6 \pm \sqrt{36 - 4 \cdot 3(-2)}}{2 \cdot 3}$

$= \frac{-6 \pm \sqrt{60}}{6}$

$x = \frac{-6 \pm 2\sqrt{15}}{6}$

$x = \frac{-3 \pm \sqrt{15}}{3}$

## Rational Equations

**Example 4** Find all real solutions to $\frac{10}{x} - \frac{12}{x-3} + 4 = 0$

$x(x-3)\left(\frac{10}{x} - \frac{12}{x-3} + 4\right) = 0$

$10(x-3) - 12x + 4x(x-3) = 0$

$10x - 30 - 12x + 4x^2 - 12x = 0$

$4x^2 - 14x - 30 = 0$

$2x^2 - 7x - 15 = 0$

$(2x + 3)(x - 5) = 0$

$2x + 3 = 0$ $\quad x = -3/2$

$x = 5$

## Radical Equations

**Example 5** Find all real solutions to the equation: $\sqrt{x+2}+x=4$

$\sqrt{x+2}^2 = (4-x)^2$

$x+2 = x^2-8x+16$

$0 = x^2-9x+14$

$0 = (x-7)(x-2)$

$x=7, x=2$

check

$x=7$ $\quad \sqrt{7+2}+7 \stackrel{?}{=} 4$ $\quad 10 \neq 4$ NO

$\boxed{x=2}$ $\quad \sqrt{2+2}+2 \stackrel{?}{=} 4$ $\quad 2+2=4$ yes.

**Example 6** Find all real solutions to the equation: $x^{1/2}+3x^{-1/2}=10x^{-3/2}$

$x^{1/2}+3x^{-1/2}-10x^{-3/2}=0$

$x^{-3/2}(x^2+3x-10)=0$

$\frac{(x+5)(x-2)}{x^{3/2}}=0$

$x=-5, 2$

$x^{1/2} = \sqrt{x}$

can't $\sqrt{}$ a negative

$\Rightarrow \boxed{x=2}$

only solution

**Example 7** Find all solution to the equation: $x^4-9x^2+20=0$

$(x^2-5)(x^2-4)=0$

$x^2-5=0$ $\quad x^2-4=0$

$\sqrt{x^2}=\sqrt{5}$ $\quad \sqrt{x^2}=\sqrt{4}$

$\boxed{x=\pm\sqrt{5} \quad x=\pm 2}$

**Example 8** Solve the absolute value equation: $2|3x+4|-10=0$

$2|3x+4|=10$

$|3x+4|=5$

$3x+4=5$ or $3x+4=-5$

$3x=1$ $\quad 3x=-9$

$x=\frac{1}{3}$ $\quad x=-3$