

## 1.4 Rational Expressions

**Objectives:** Find the domain of expressions; simplify rational expressions; factor rational expressions.

The **domain** of an expression is the set of all possible values the variable can have.

**Example 1** Find the domain for the following expressions.

(a)  $\frac{x-4}{x^2-4x-77}$

$$x^2 - 4x - 77 \neq 0$$

$$(x+7)(x-11) \neq 0$$

$$x \neq -7 \text{ or } x \neq 11$$

$$D: (-\infty, -7) \cup (-7, 11) \cup (11, \infty)$$

Simplifying Expressions

**Example 2** Simplify:  $\frac{x^2-x-6}{x^2+2x} \cdot \frac{x+1}{x^2-2x-3}$

$$\frac{(x-3)(x+2)}{x(x+2)} \cdot \frac{x+1}{(x-3)(x+1)}$$

$$\boxed{\frac{1}{x}}$$

**Example 3** Simplify:  $\frac{2x^3+16}{x^2-2x-15} \div \frac{x^2-2x+4}{x^2-25}$

$$\frac{2(x^3+8)}{x^2-2x-15} \cdot \frac{x^2-25}{x^2-2x+4}$$

$$\frac{2(x+2)(x^2-2x+4)}{(x-5)(x+3)} \cdot \frac{(x-5)(x+5)}{(x^2-2x+4)}$$

**Example 4** Simplify:  $\frac{13}{5x-20} - \frac{3}{5x+5}$

$$\frac{13}{5(x-4)} - \frac{3}{5(x+1)}$$

$$\frac{13(x+1)}{5(x-1)(x+1)} - \frac{3(x-4)}{5(x+1)(x-4)}$$

(b)  $\frac{\sqrt{x}}{2x-8} \rightarrow x \geq 0$

$$2x-8 \neq 0$$

$$x \neq 4$$



$$D: [0, 4) \cup (4, \infty)$$

$$\boxed{\frac{2(x+2)(x+5)}{x+3}}$$

$$\frac{13x+13 - 3x+12}{5(x-4)(x+1)}$$

$$5(x-4)(x+1)$$

$$10x+25$$

$$5(x-4)(x+1)$$

$$\frac{10x+25}{5(x-4)(x+1)}$$

$$\frac{2x+5}{(x-4)(x+1)}$$

$$\boxed{\frac{2x+5}{(x-4)(x+1)}}$$

**Example 5**Simplify:  $\frac{\frac{3}{x} - \frac{4}{x+1}}{\frac{2}{x+1} + \frac{1}{2x}}$ 

LCD =  $2x(x+1)$

$$\left( \frac{3}{x} - \frac{4}{x+1} \right) 2x(x+1)$$

$$\left( \frac{2}{x+1} + \frac{1}{2x} \right) 2x(x+1)$$

$$\frac{6(x+1) - 8x}{4x + (x+1)}$$

$$\frac{6x + 6 - 8x}{4x + x + 1}$$

$$\boxed{\frac{-2x + 6}{5x + 1}}$$

or 
$$\frac{-2(x-3)}{5x+1}$$

**Example 6**Simplify the expression by factoring the numerator:  $\frac{2(x+1)(3x+1)^3 - (x+1)^2(3x+1)^2}{(3x+1)^3}$ . Expressions like thisarise from the use of the *quotient rule* in Calculus II.

$$\frac{2(x+1)(3x+1)^3 - 6(x+1)^2(3x+1)^2}{(3x+1)^6}$$

$$= \frac{2(x+1)(3x+1)^2 [3x+1 - 3(x+1)]}{(3x+1)^6}$$

$$= \frac{2(x+1)(3x+1-3x-3)}{(3x+1)^4}$$

$$= \frac{2(x+1)(-2)}{(3x+1)^4}$$
  
$$= \boxed{\frac{-4(x+1)}{(3x+1)^4}}$$

**Example 7**Simplify:  $\frac{\frac{3}{2}(x-5)^{-1/2}(3x) - 3(x-5)^{1/2}}{(3x)^2}$ 

$$\frac{\frac{3}{2}(x-5)^{-1/2} [x - 2(x-5)]}{9x^2}$$

$$\frac{\cancel{3} (x - 2x + 10)}{2(x-5)^{1/2} \cdot \frac{9}{\cancel{3}} x^2}$$

$$\boxed{\frac{-x + 10}{6x^2(x-5)^{1/2}}}$$

or 
$$\frac{-(x-10)}{6x^2(x-5)^{1/2}}$$