1.3 Algebraic Expressions

Objectives: Be able to simplify expressions; factor trinomials; use special factoring formulas; factor by grouping.

Simplifying Expressions

Example 1 Simplify the expression: $(xy - y^2)(x^2 + xy + y^2)$

$$x^3y + x^2y^2 + xy^3 - y^2x^2 - xy^3 - y^4$$

$$\boxed{x^3y - y^4}$$

Example 2 Simplify: $\left(\sqrt{h^2 + 1} + 1\right)\left(\sqrt{h^2 + 1} - 1\right)$

$$h^2 - 1 \quad -1$$

$$\boxed{h^2}$$

Factoring Expressions

Example 3 Factor the trinomials:

(a) $x^2 - 10x - 24$

$$(x + 2)(x - 12)$$

(b) $10x^2 + 29x - 21$

$$(5x - 3)(2x + 7)$$

(c) $x^4 + 3x^2 - 28$

$$(x^2 + 7)(x^2 - 4)$$

$$(x^2 + 7)(x - 2)(x + 2)$$

Example 4 Factor: $3x^3 + 5x^2 - 6x - 10$

$$x^2(3x + 5) - 2(3x + 5)$$

$$(3x + 5)(x^2 - 2)$$

Example 5   Factor: $12 x^3 y^5 + 8 x^5 y^4$

$$4x^3 y^4 \left(3y + 2x^2 \right)$$

Example 6   Factor: $2 x^{-1} y + 10 x^{-2} y^2 + 12 x^{-3} y^3$

$$2 x^{-3} y \left(x^2 + 5xy + 6y^2 \right)$$

$$\frac{2y (x + 3y)(x + 2y)}{x^3}$$

The next two factoring examples are expressions that result from *The Product Rule* used in Calculus I.

Example 7   Factor: $4 (2x + 5)^3 \, 2 (x + 4)^6 + (2x + 5)^4 \, 6 (x + 4)^5$

$$8 (2x+5)^3 (x+4)^6 + 6 (2x+5)^4 (x+4)^5$$

$$2 (2x+5)^3 (x+4)^5 \left[4(x+4) + 3(2x+5) \right]$$

$$2 (2x+5)^3 (x+4)^5 \left(4x + 16 + 6x + 15 \right)$$

$$2 (2x+5)^3 (x+4)^5 \left(10x + 31 \right)$$

Example 8   Factor: $3 x^2 (2x + 1)^{1/2} + x^3 \cdot \frac{1}{2} (2x + 1)^{-1/2} \cdot 2$

$$3x^2 (2x+1)^{\frac{1}{2}} + x^3 (2x+1)^{-\frac{1}{2}}$$

$$x^2 (2x+1)^{-\frac{1}{2}} \left[3(2x+1) + x \right]$$

$$\frac{x^2 (6x + 3 + x)}{(2x+1)^{\frac{1}{2}}}$$

$$\frac{x^2 (7x+3)}{(2x+1)^{\frac{1}{2}}}$$