Math 141 Pre-Calc I In-Class Notes

1.2 Review of Exponents and Radicals

Objectives: Use the properties of exponents; use rational exponents; use scientific notation; simplify radicals.

Exponential Notation

Integer exponents are used for repeated multiplication: $a^n = \underbrace{a \cdot a \cdots a}_{n\text{ factors}}$, a is the **base** and n is the **exponent**.

Laws of Exponents

1. $a^0 = 1$
2. $a^{-n} = \frac{1}{n}$
3. $a^m a^n = a^{m+n}$
4. $\frac{a^m}{a^n} = a^{m-n}$
5. $(a^m)^n = a^{mn}$
6. $(a\,b)^n = a^n b^n$
7. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
8. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
9. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

$(a+b)^n \neq a^n + b^n$!!

Example 1 Simplify: $\frac{(2x^2)^3\, 3x^4}{(x^3)^5}$

$\frac{8x^6 \cdot 3x^4}{x^{15}}$

$\frac{24x^{10}}{x^{15}}$

$\frac{24}{x^5}$

Example 2 Simplify: $(3\,a\,b^2\,c)\left(\frac{2a^{-2}b}{c^3}\right)^{-2}$

$3ab^2c\left(\frac{c^3}{2a^{-2}b}\right)^2$

$3ab^2c\,\frac{c^6}{4a^{-4}b^2}$

$\frac{3a\cancel{b^2}c^7}{4a^{-4}\cancel{b^2}}$

$\frac{3a^5c^7}{4}$

Scientific Notation

Scientific notation is used to write extremely large numbers or extremely small numbers in a more useful form. For instance, the number of molecules in a cup of water is approximately 8,360,000,000,000,000,000,000,000; the mass of a neutron is 0.00000 00000 00000 00000 00000 016749 kg. In scientific notation these numbers are

$$8{,}360{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 8.36 \times 10^{24}$$
$$0.000000000000000000000000016749 = 1.6749 \times 10^{-27}\text{ kg}$$

Example 3 Let $a = 0.00000361$, $b = 2.43 \times 10^{12}$, and $c = 3.27 \times 10^{-18}$. **(a)** Write a in scientific notation and **(b)** calculate $\frac{b}{ac}$.

$a = 3.61 \times 10^{-6}$

$\frac{b}{ac} = \frac{(2.43 \times 10^{12})}{(3.61 \times 10^{-6})(3.27 \times 10^{-18})}$

$= \frac{2.43}{(3.61)(3.27)} \times 10^{12}\,10^{6}\,10^{18}$

$\to 0.206 \times 10^{36}$

$\boxed{2.06 \times 10^{35}}$

Radicals and n^{th} roots

If n is any positive integer, then the **principal n^{th} root of a** is defined as $\sqrt[n]{a} = b$ meaning $b^n = a$. If n is even then $a \geq 0$ and $b \geq 0$.

Properties of nth Roots

1. $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$
2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
4. $\sqrt[n]{a^n} = a$ if n is odd
5. $\sqrt[n]{a^n} = |\,a\,|$ if n is even

Example 4 Simplify: **(a)** $\sqrt[3]{-27x^6y^9}$ **(b)** $\sqrt[4]{16x^4y^8z^{16}}$

$-3x^2y^3$

$2|x|\,y^2z^4$

Example 5 Simplify: $x\sqrt{27x} + 2\sqrt{75x^3}$

$x\sqrt{9\cdot 3x} + 2\sqrt{25x^2\cdot 3x}$

$3x\sqrt{3x} + 10x\sqrt{3x}$

$13x\sqrt{3x}$

Rational Exponents

By definition, $a^{1/n} = \sqrt[n]{a}$, for the rules of exponents and radicals to apply. In general, we have the following:

Definition of Rational Exponents

For any rational exponent m/n in lowest terms with $n > 0$, we define

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} \quad \text{If } n \text{ is even, then } a \geq 0$$

⌘ Calculator Tip: Since the cube-root of number can be written as a^(1/3), and also $1/3 = 3^{-1}$, the cube root of 17 can be calculated as 17^3^{-1}. On your calculator, you may have an x^{-1} button that can easily be used to calculate roots. For example, $\sqrt[5]{20} = 20$^5 x^{-1} ENTER.

Example 6 Solve the equations: **(a)** $x^4 = 80$ **(b)** $x^{2.432} = 20$

$x^4 = 80$

$x = \sqrt[4]{80}$

$x = 80^{(1/4)}$

$x \approx 2.9907$

$x = 20^{\left(\frac{1}{2.432}\right)}$

$x = 3.4274$

Example 7 Simplify: **(a)** $(2x^{3/2})(4x)^{-1/2}$ **(b)** $\left(\frac{a^2b^{-3}}{x^{-1}y^2}\right)\left(\frac{x^{-2}b^{-1}}{a^{3/2}y^{1/3}}\right)$ **(c)** $\sqrt[3]{x^2}\cdot\sqrt[4]{x^3}$

$2x^{3/2}\cdot 4^{-1/2}x^{-1/2}$

$\frac{2x}{4^{1/2}} = \frac{2x}{2}$

$= x$

$\frac{a^2b^{-3}x^{-2}b^{-1}}{x^{-1}y^2a^{3/2}y^{1/3}} = \frac{a^2x}{b^3x^2by^2a^{3/2}y^{1/3}}$

$= \frac{a^{1/2}}{b^4xy^{1/3}}$

$x^{2/3}\cdot x^{3/4}$

$x^{\frac{2}{3}+\frac{3}{4}}$

$x^{\frac{8}{12}+\frac{9}{12}}$

$x^{17/12}$ or $x\sqrt[12]{x^5}$