

9.8 Partial Fractions

Example 1 Combine and simplify the expression: $\frac{4}{2x+3} + \frac{5}{x-5}$

Partial fraction decomposition is the process of undoing the combining of rational fractions. For example, the partial decomposition of $\frac{6x-14}{x^2-2x-15}$ is $\frac{4}{x+3} + \frac{2}{x-5}$, since

$$\begin{aligned}\frac{4}{x+3} + \frac{2}{x-5} &= \frac{4}{x+3} \cdot \frac{x-5}{x-5} + \frac{2}{x-5} \cdot \frac{x+3}{x+3} \\ &= \frac{4x-20}{(x+3)(x-5)} + \frac{2x+6}{(x+3)(x-5)} \\ &= \frac{6x-14}{x^2-2x-15} \quad \blacksquare\end{aligned}$$

Example 2 Find A and B such that: $\frac{3x-23}{x^2+x-12} = \frac{A}{x+4} + \frac{B}{x-3}$. (Multiply by the LCD and create a system of equations to solve for A and B .)

Rules for Decomposing Rational Expressions

1. Distinct linear factors: $\frac{P(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$
2. Distinct irreducible quadratic factors: $\frac{P(x)}{(ax^2+bx+c)(dx^2+ex+f)} = \frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{dx^2+ex+f}$
3. Distinct linear and irreducible quadratic factors: $\frac{P(x)}{(ax+b)(cx^2+dx+e)} = \frac{A}{ax+b} + \frac{Bx+C}{cx^2+dx+e}$
4. Repeated factors: $\frac{P(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \dots + \frac{N}{(ax+b)^n}$

(Note: if the degree of the numerator is equal to or greater than the denominator degree, long divide first.)

Example 3 Set up the required partial fractions for: $\frac{3x^2-2x+1}{(x+1)^3(x^2+2x+4)^2}$.

Example 4 Find the partial fraction decomposition for: $\frac{11x^2 - 3x + 4}{(2x+1)(x^2+x+3)}$.

Example 5 Find the partial fraction decomposition for: $\frac{3x-5}{2x^2+x-3}$.

Shortcut Method

Often, a shortcut can be made when finding the unknown parameters. After multiplying by the common denominator, evaluate the equation with the zeros of the denominator.

Example 6 Use the shortcut method to help find the partial fraction decomposition for: $\frac{3x+1}{x^2-7x+12}$.

Example 7 Find the partial fraction decomposition for: $\frac{11x^2-4x+16}{2x^3-4x^2+5x-10}$.

Example 8 Decompose: $\frac{5}{x^4-16}$.

Example 9 Decompose: $\frac{10x^4-35x^3+63x^2-85x+63}{x^5-5x^4+10x^3-18x^2+21x-9}$. (Find the zeros of the denominator to get it factored.)