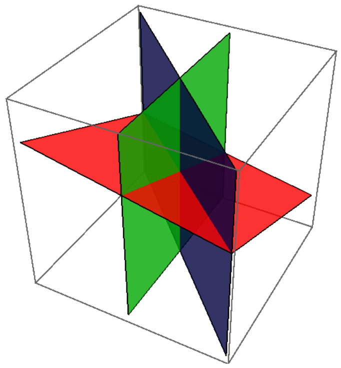
9.3 Systems of Linear Equations in Several Variables

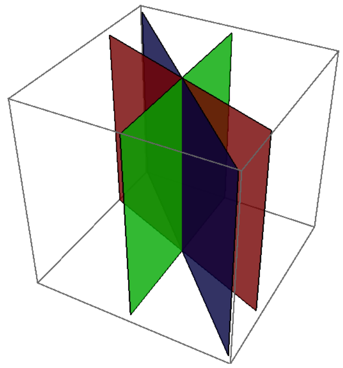
A system of linear equations can have either one solution, no solutions, or infinitely many solutions. Some possibilities for systems of equation in three variables are shown below.

One Solution



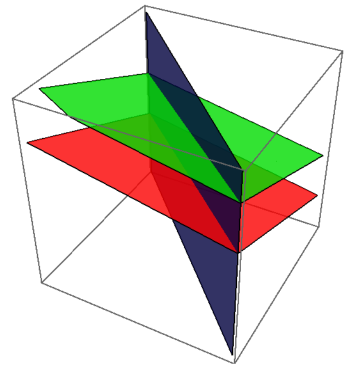
Consistent
Independent

Infinite Solutions



Consistent
Dependent

No Solutions



Inconsistent

The Elimination Method

One method used to solve a system of linear equations is the elimination method

Step 1 Choose two equations and eliminate a variable.

Step 2 Choose a different pair of equations and eliminate the same variable as in step 1.

Step 3 Solve the remaining two-variable two-equation system.

Step 4 Back-substitute to solve for the third variable.

Example 1 Solve the system of equations, or show that it is inconsistent.

$$\begin{cases} 2x + 4y - z = 2 \\ x + 2y - 3z = -4 \\ 3x - y + z = 1 \end{cases}$$

Example 2 Solve the system of equations, or show that it is inconsistent.

$$\begin{cases} x - 2y - 3z = 5 \\ 2x + y - z = 5 \\ 4x - 3y - 7z = 5 \end{cases}$$

Example 3 Use matrices to solve the systems in examples 1 and 2. First enter coefficients of the system into matrix **[A]**, and then use the **RREF** command to reduce the matrix to *reduced row echelon form*, and write the solution.

Systems with Infinite Solutions

A system of linear equations that is consistent and dependent can have its solution set written in parametric from.

Example 4 Find the solution to the system of dependent equations:

$$\begin{cases} x + 2y - 3z = 1 \\ x - y + 2z = 4 \\ 2x + y - z = 5 \end{cases}$$

Example 5 Find the equation of the line of intersection of the two planes $3x + 2y + z = 6$, and $4x + 3y - z = 7$.

Example 6 Find the equation of a quadratic whose graph passes through the points (−2, 3), (1, 2), and (3, 8).