4.5 Modeling with Exponential and Logarithmic Functions

Example 1 ***Unlimited Growth***

The population of a small town in 1970 was 10,675. By 1985 the population had grown to 20,043. Find an exponential model in the form $P(t) = P_0\,e^{rt}$ for the population, and predict the population for the year 2000. What year will the population exceed 50,000 assuming a constant rate of growth.

Example 2 ***Percent Increase/Decrease***

The value of a $23500 automobile decreases about 20% every 3 years. Find a model in the form $V(t) = V_0\,b^{t/k}$ and estimate the time it takes for the value to be $10000.

Example 3 ***Radioactive Decay***

Radioactive Carbon-14 has a half-life of 5730 years. A bone fragment was found to contain only 27.6% of its original C-14. Find an exponential function for the amount of remaining C-14 and estimate the age of the bone fragment.

Example 4 ***Compound Interest***

If a lump sum of money, *PV (present value)* , is deposited into an account with annual interest r compounded n times per year for t years, the future value (FV) of the account is given by: $FV = PV\left(1 + \frac{r}{n}\right)^{nt}$. Suppose \$8000 is deposited into an account at 5% interest compounded monthly.

(a) Find a function for the future value of the account after t years.
(b) Estimate the future value after 10 years.
(c) How long will it take for the account to double its initial value.

Example 5 ***Newton's Law of Cooling***

Newton's Law of Cooling states the rate an object cools is proportional to the difference in the initial temperature T_o, and the surrounding temperature T_s. The temperature of the object at any time t is $T(t) = (T_0 - T_s)\,e^{-kt} + T_s$.
Suppose a piece of iron initially at 1500 °C cools to 1100 °C after 30 minutes in a 16 °C room.

(a) Find a function for the temperature at time t.
(b) Estimate the temperature after 3 hours.
(c) How long will it take for the iron to be less than 50 °C?

Example 6 ***The Logistic Formula***

The logistic formula, $f(t) = \frac{L}{1 + a\,e^{-rt}}$ can be use when the size of a population is limited (not unbounded). Suppose a population is modeled by the logistic growth function $P(t) = \dfrac{32\,000}{1 + 49\,e^{-0.045\,t}}$ where t is in years.

(a) Find the initial population at time $t = 0$.
(b) Find the limiting population as $t \to \infty$.
(c) Find the population after 40 years.
(d) The population is growing at its greatest rate when $P = \frac{L}{2}$. At what time t does this happen?