

## 4.5 Modeling with Exponential and Logarithmic Functions

### Example 1 *Unlimited Growth*

The population of a small town in 1970 was 10,675. By 1985 the population had grown to 20,043. Find an exponential model in the form  $P(t) = P_0 e^{rt}$  for the population, and predict the population for the year 2000. What year will the population exceed 50,000 assuming a constant rate of growth.

### Example 2 *Percent Increase/Decrease*

The value of a \$23500 automobile decreases about 20% every 3 years. Find a model in the form  $V(t) = V_0 b^{t/k}$  and estimate the time it takes for the value to be \$10000.

### Example 3 *Newton's Law of Cooling*

Newton's Law of Cooling states the rate an object cools is proportional to the difference in the initial temperature  $T_0$ , and the surrounding temperature  $T_s$ . The temperature of the object at any time  $t$  is  $T(t) = (T_0 - T_s) e^{-kt} + T_s$ . Suppose a piece of iron initially at 1500 °C cools to 1100 °C after 30 minutes in a 16 °C room.

- Find a function for the temperature at time  $t$ .
- Estimate the temperature after 3 hours.
- How long will it take for the iron to be less than 50 °C?

**Example 4** *Radioactive Decay*

Radioactive Carbon-14 has a half-life of 5730 years. A bone fragment was found to contain only 27.6% of its original C-14. Find an exponential function for the amount of remaining C-14 and estimate the age of the bone fragment.

**Example 6** *The Logistic Formula*

The logistic formula,  $f(t) = \frac{L}{1+ae^{-rt}}$  can be use when the size of a population is limited (not unbounded). Suppose a population is modeled by the logistic growth function  $P(t) = \frac{32000}{1+49e^{-0.045t}}$  where  $t$  is in years.

- Find the initial population at time  $t = 0$ .
- Find the limiting population as  $t \rightarrow \infty$ .
- Find the population after 40 years.
- The population is growing at its greatest rate when  $P = \frac{L}{2}$ . At what time  $t$  does this happen?