4.4 Solving Exponential and Logarithmic Equations

Steps for Solving an Exponential Equation

1. Isolate the exponential term.
2. Take the logarithm of each side (usually base e or base 10)
3. Solve for the variable
4. Check your answer.

Example: Solve the equation: $300 = 5 \cdot 1.2^x$.

Step 1: $\frac{300}{5} = 1.2^x \implies 60 = 1.2^x$ *Divide each side by 5.*

Step 2: $\ln(60) = \ln(1.2^x) \implies \ln(60) = x \ln(1.2)$ *Take the natural log of each side.*

Step 3: $\frac{\ln(60)}{\ln(1.2)} = x \implies x \approx 22.4567$ *Divide by ln(1.2).*

Step 4: Check: $5 \times 1.2^{22.4567} \approx 299.999 \approx 300$ *Check your answer.*

Example 1 Solve the equation: $2 \cdot 4^x - 5 = 23$

Example 2 Solve the equation: $57 = 300\, e^{-0.085\, t}$

Example 3 Solve the equation: $2^{3x+4} = 3^{2x-1}$

Steps for Solving Logarithmic Equations

1. Get all the logarithm terms on one side of the equation.
2. Combine log terms into a single logarithm.
3. Exponentiate each side to eliminate the logarithm (or convert to exponential form).
4. Solve for the variable.
5. Check your answer.

Example: Solve the equation $\log_3(x+3) = 2 - \log_3(x-5)$

Step 1: $\log_3(x+3) + \log_3(x-5) = 2$

Step 2: $\log_3((x+3)(x-5)) = 2 \implies \log_3\left(x^2 - 2x - 15\right) = 2$

Step 3: $3^{\log_3(x^2-2x-15)} = 3^2 \implies x^2 - 2x - 15 = 9$

Step 4: $x^2 - 2x - 24 = 0 \implies (x-6)(x+4) = 0 \implies x = 6$ or $x = -4$

Step 5: $x = 6$ works, but not -4 since you can't log a negative number.

Example 4 Solve the equation $\log_5(x-4) + 2 = \log_5(2x+7)$

Example 5 Solve the equation $\log_6(x+4) = 1 - \log_6(x+3)$

Example 6 Solve: $\frac{e^x + e^{-x}}{2} = 2$

Example 7 Solve for t: $200 = \frac{500}{1+4e^{-0.5t}}$