4.3 Laws of Logarithms

The proofs of these properties are shown in class.

Numerical Properties

1. $\log_b(1) =$
2. $\log_b(b) =$

Inverse Properties

3. $\log_b(b^x) =$
4. $b^{\log_b(x)} =$

Example 1 Simplify the following:

(a) $\log_7(7^5)$ (b) $\ln(e^{-0.254})$ (c) $10^{\log(\pi)}$

Algebraic Properties

5. $\log_b(x\,y) = \log_b(x) + \log_b(y)$
6. $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
7. $\log_b(x^n) = n\log_b(x)$

Example 2 Fully expand the expression: $\log_4\left(\frac{3x^2}{5y}\right)$.

Example 3 Fully expand and simplify: $\log_6\left(\frac{\sqrt{6x}}{7y^3}\right)$.

Example 4 Write as a single logarithm: $2\log_3(x) - 3\log_3(2) + \frac{1}{2}\log_3(y) - 4\log_3(z)$

Example 5 Evaluate $\frac{3\ln(2)+4\ln(3)}{2\ln(3)-5\ln(2)}$ on your calculator, then use the properties of logarithms to make a simpler expression. Use your calculator to check your answer.

Conversion Formulas

8. $b^x = e^{x\ln(b)}$ (Convert from base b to e) *Example*: $1.05^x = e^{\ln(1.05^x)} = e^{x\ln(1.05)} = e^{0.0488x}$

9. $\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$ (**Change of base formula**) *Example*: $\log_5(38) = \frac{\ln(38)}{\ln(5)}$ or $\frac{\log(38)}{\log(5)} \approx 2.2602$

Example 6 Water consumption in a rural area is increasing 15% every 4 years. If the current usage is 15.6 million cubic feet per year the exponential model for water consumption is $W(t) = 15.6\,(1.15)^{t/4}$. Rewrite the water usage model using base e, and use both models to predict the usage in 10 years.

Example 7 Use the properties of logarithms and the change of base formula to solve the equation: $4^x = 20$.

Example 8 Graph the logarithmic function $f(x) = \log_3(x)$.