4.2 Logarithmic Functions

From section 4.1 we can see that all exponential functions are one-to-one which means they have an inverse. Let's find the inverse.

0. $f(x) = b^x$ is one-to-one
1. $y = b^x$
2. $x = b^y$
3. Solve for y. How?!
4. $f^{-1}(x) = ?$

With our current functions, we have no way to solve for y. This can be resolved by using logarithm functions**.**

Logarithms

Let $x = b^y$ be an exponential expression with $b > 0$ and $b \neq 1$. Then, the **logarithm base *b* of *x* is equal to *y***. Or,

> If $x = b^y$, then $\log_b(x) = y$

This means in step (3) above we have:

3. $y = \log_b(x)$
4. $f^{-1}(x) = \log_b(x)$ ■

Example 1 Rewrite the following exponential expressions into logarithmic expressions:

(a) $2^5 = 32$ (b) $3^{-2} = \frac{1}{9}$ (c) $\left(\frac{1}{5}\right)^3 = \frac{1}{125}$ (d) $\left(\frac{1}{4}\right)^{-3} = 64$

Example 2 Rewrite the logarithmic expression into exponential form:

(a) $\log_2(8) = 3$ (b) $\log_5\left(\frac{1}{25}\right) = -2$ (c) $\log_7(1) = 0$

Example 3 Find the missing value:

(a) $\log_3(81) = x$ (b) $\log_b(17) = 2$ (c) $\log_{1/2}(x) = -4$ (d) $\log_b(14) \approx 1.63974$

Example 4 Base 4 raised to the power of 3 equals the number 64. Write these three numbers mathematically where the 4 is isolated; the 3 is isolated; and the 64 is isolated.

Example 5 Graph the exponential function $f(x) = 2^x$ plotting several points and the horizontal asymptote. Find the inverse function $f^{-1}(x)$ plotting several points and the asymptote. Write down the domain and range of f as well as the domain and range of f^{-1}.

The Common Logarithm

Our common counting system is base 10, so using a logarithm with base 10 is called the *common logarithm*

$$\log_{10}(x) = \log(x)$$

Example 6 Find log(10 000) by hand and using your calculator.

Example 7 Use your calculator and logarithms to solve the exponential equation: $10^x = 600$ to four decimal places.

Example 8 Solve: $10^{3x-5} = 72$ to four decimal places.

The Natural Logarithm

When the natural number e is used as the base of a logarithm we call it the *natural logarithm*

$$\log_e(x) = \ln(x)$$

Example 9 Find ln(8) to four decimal places and rewrite the expression in exponential form.

Example 10 Solve the equation: $e^{3x} = 100$ to four decimal places and check your answer.