

## 4.1 Exponential Functions

An exponential function is any function that can be written



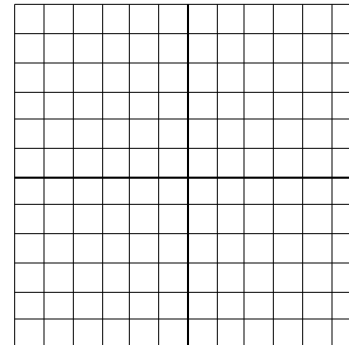
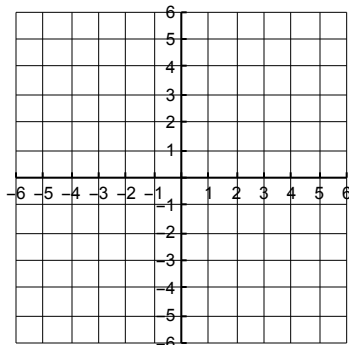
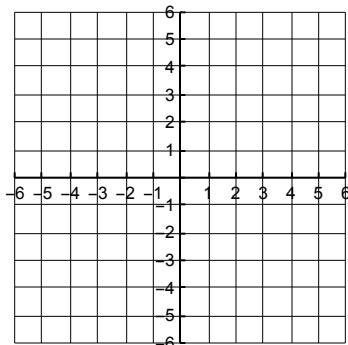
$$f(x) = b^x \text{ where } b > 0 \text{ and } b \neq 1$$

### Example 1

Graph the functions: a)  $f(x) = 2^x$

b)  $g(h) = \left(\frac{1}{2}\right)^x$

c)  $h(x) = 5 \cdot 2^{x/3}$



### Example 2

Use transformations to graph the function  $f(x) = -3^{x-4} + 2$ .

## Relative Growth Rate Exponential Function

$$A(t) = A_0(1 \pm r)^{t/K}$$

where,  $A_0$  is the initial amount,  $r$  is the relative growth (or decay rate if  $r < 0$ ) rate,  $K$  is the length of time to achieve the growth, and  $t$  is time.

*Example:* The population of rabbits in an area increases 20% every 8 months. If there are initially 200 rabbits, a model for the population is

$$P(t) = 200(1.2)^{t/8} \text{ where } t \text{ is in months.}$$

### Example 3

Many vehicles depreciate 20% of their value about every three years. Estimate the value of a \$35000 vehicle after 10 years.

**Example 4** **Compound Interest** is calculated using the formula  $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$  where  $A$  is the future value,  $P$  is the principal,  $r$  is the annual interest rate, and  $n$  is the number of compoundings per year. Calculate the future value of a \$1000 deposit at 6% annual interest rate compounded yearly, monthly, and daily, after 2 years.

**Example 5** Use your calculator to explore numerically and graphically what happens to the expression  $\left(1 + \frac{1}{x}\right)^x$  as  $x \rightarrow \infty$ .

## The Natural Exponential Function

$$A(t) = P e^{rt}$$

where  $e$  is the natural number  $e \approx 2.71828182845904 \dots$ , and  $r$  is the continuous growth rate.

**Example 6** A town had a population of 23,750 people in 2005 and a continuous rate of growth of 3.28%. Write the exponential function that models the population and estimate the population in 2020.

**Example 7** **Continuous Compound Interest** uses  $A(t) = P e^{rt}$  where  $A$  is the future value,  $P$  the principal,  $r$  is the annual interest rate, and  $t$  is number of years. Calculate the future value of the \$1000 deposit at 6% interest after 2 years compounded continuously.