

4.1 Exponential Functions

An exponential function is any function that can be written



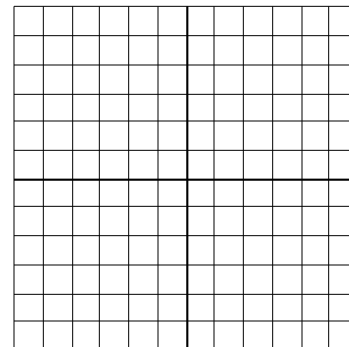
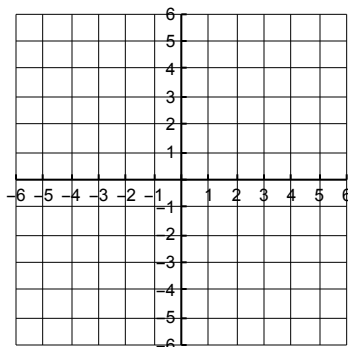
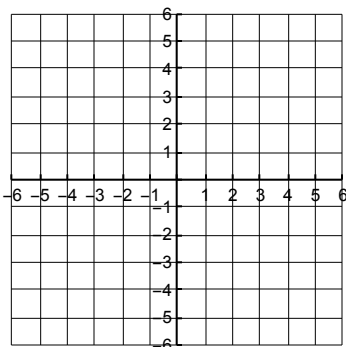
$$f(x) = b^x \text{ where } b > 0 \text{ and } b \neq 1$$

Example 1

Graph the functions: a) $f(x) = 2^x$

b) $g(h) = \left(\frac{1}{2}\right)^x$

c) $h(x) = 5 \cdot 2^{x/3}$



Example 2

Use transformations to graph the function $f(x) = -\left(\frac{1}{2}\right)^{x-4} + 3$.

Relative Growth Rate Exponential Function

$$A(t) = A_0(1 \pm r)^{t/K}$$

where, A_0 is the initial amount, r is the relative growth (or decay rate if $r < 0$) rate, K is the length of time to achieve the growth, and t is time.

Example: The population of rabbits in an area increases 20% every 8 months. If there are initially 200 rabbits, a model for the population is

$$P(t) = 200(1.2)^{t/8} \text{ where } t \text{ is in months.}$$

Example 3

Many vehicles depreciate 20% of their value about every three years. Estimate the value of a \$35000 vehicle after 10 years.

Example 4 **Compound Interest** is calculated using the formula $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ where A is the future value, P is the principal, r is the annual interest rate, and n is the number of compoundings per year. Calculate the future value of a \$1000 deposit at 6% annual interest rate compounded yearly, monthly, and daily, after 2 years.

Example 5 Use your calculator to explore numerically and graphically what happens to the expression $\left(1 + \frac{1}{x}\right)^x$ as $x \rightarrow \infty$.

The Natural Exponential Function

$$A(t) = P e^{rt}$$

where e is the natural number $e \approx 2.71828182845904 \dots$, and r is the continuous growth rate.

Example 6 A town had a population of 23,750 people in 2005 and a continuous rate of growth of 3.28%. Write the exponential function that models the population and estimate the population in 2020.

Example 7 **Continuous Compound Interest** uses $A(t) = P e^{rt}$ where A is the future value, P the principal, r is the annual interest rate, and t is number of years. Calculate the future value of the \$1000 deposit at 6% interest after 2 years compounded continuously.