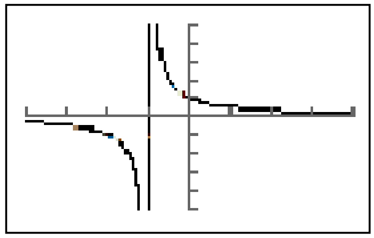
# 3.6 More Properties of Rational Functions

**Objectives**: Analyze the graph of a rational function; find a hole in a rational function; solved applied problems.

## Common Factors

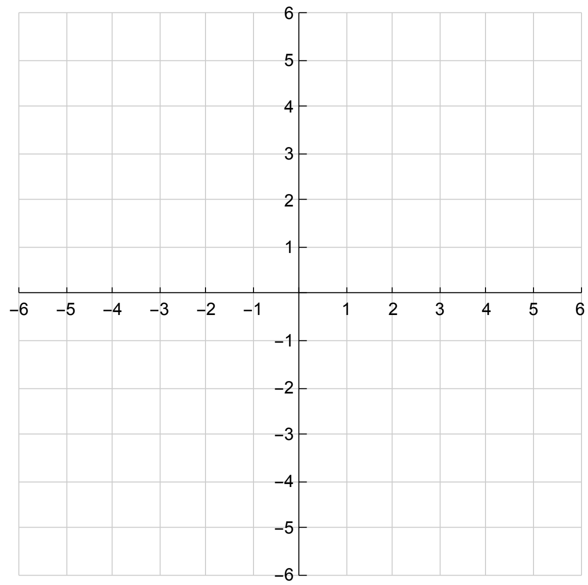
Suppose the rational function $f(x) = \frac{p(x)}{q(x)}$ have a common factor in the numerator and denominator. What happens?

**Example 1** Use your calculator to graph the function $f(x) = \frac{x-1}{x^2-1}$. Note, some calculators may accidentally connect the parts of the graph where the asymptotes is. This should not be there.



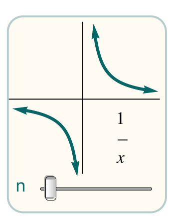
What is *really* happening at $x = 1$? If you press **[ZOOM]** **4: ZoomDec** you'll get a better idea.

**Example 2** Make an accurate graph of $f(x) = \frac{x^2+3x-4}{x^2-3x+2}$.

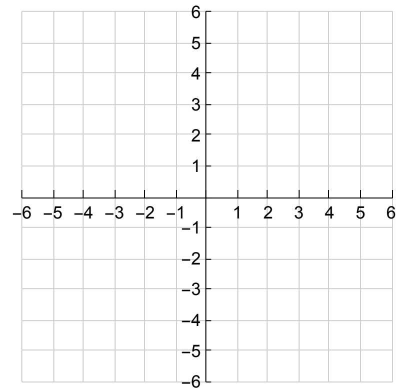



## Multiplicities of the Asymptote Factors

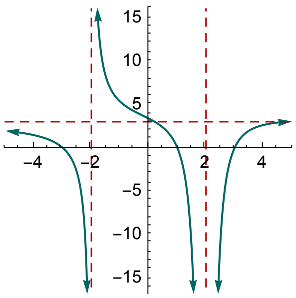
Recall the graph of the toolkit function $f(x) = \frac{1}{x}$. Compare the graphs of $\frac{1}{x^2}$, $\frac{1}{x^3}$, $\frac{1}{x^4}$, $\frac{1}{x^5}$, etc. Notice the similarities with the multiplicities of the zeros for polynomials?



**Example 3** Make a sketch the graph of $f(x) = \dfrac{x-1}{(x+1)\,(x-2)^2}$




**Example 4** Find a function whose graph is given.




## Advanced Analysis of Rational Functions

The last feature we can check is if the function ever crosses a horizontal asymptote. This can reveal an unseen local extrema.

**Example 5** Analyze and make a graph the given function indicating all intercepts, asymptotes, holes, and any other important features. (Does the function cross the horizontal asymptote?)

$f(x) = \dfrac{2x^2 - 2x}{x^2 + x - 6}$