3.5 Complex Zeros of a Polynomial

Objectives: Find the complex zeros of a polynomial; create a polynomial from specified zeros.

Example 1 Find the zeros of $f(x) = x^3 - 8x^2 + 25x - 26$ given one zero is $x = 2$.

The zeros $3 - 2i$ and $3 + 2i$, are called **complex conjugates**. This leads to our first theorem (not proved here).

Theorem 1 Conjugate Zeros Theorem

If $f(x)$ is a polynomial with **real** coefficients, and z is a *complex zero* of f, then the conjugate $\bar{z}$ is also a zero. This means if f has real coefficients, and $z_1 = a + bi$ is a zero then $z_2 = a - bi$ is also a zero.

Example 2 Find the zeros to the polynomial $f(x) = x^3 - 7x^2 + 25x - 39$.

NOTE However, if f has *complex coefficients*, the zeros may **not** occur in conjugate pairs.

Example 3 For the quadratic $f(x) = x^2 - (5 + i)x + 6 + 3i$ use synthetic division to show that $2 + i$ is a zero but its conjugate $2 - i$ is **not** a zero.

Example 4 Use the graph of the polynomial to find **all** the zeros: $f(x) = 2x^5 - 3x^4 - 22x^3 + 2x^2 + 96x + 45$.

Theorem 2 Fundamental Theorem of Algebra

If f is a polynomial of degree n with complex coefficients, then f has at least one complex zero. (Remember, real numbers are also complex numbers.)

We won't prove this, but it leads to the next theorem.

Theorem 3

Every polynomial function f with real coefficients of degree $n \geq 1$ can be factored into a product of linear and irreducible quadratic factors with real coefficients.

Example 5 Use *Descartes' Rule of Signs* to list **all** possible combinations of zeros for the given polynomial; list all possible *rational* zeros, and then find the zeros. Hint: use your calculator to identify the rational zeros.
$f(x) = 3x^5 + 2x^4 - 7x^3 + 14x^2 - 76x + 24$

Example 6 A polynomial with real coefficients has zeros: -3; 1; and $2 - i$. Find the polynomial.

Example 7 Find all the solutions to the equation: $4x^4 + 2x^3 + 1 = 2x^2 + 3x + 2$.

Example 8 Find all the solutions to $x^2 + 2ix - 1 = 0$.