3.2 Dividing Polynomials

Division of polynomials is similar to the long division of integers. For instance, suppose we divide 5347 by 21. The result is 254 with a remainder of 13. We can express this result as

$$\frac{5347}{21} = 254 + \frac{13}{21}$$

We can also multiply each side by 21 to get: $5347 = 21 \times 254 + 13$. **Terminology**: 5347 is called the *dividend*, 21 is the *divisor*, 254 is the *quotient*, and 13 is the *remainder.*

Example 1 Use long division to divide $4x^2 - 5x + 12$ by $x - 3$ and identify the dividend, divisor, quotient and remainder.

Division Algorithm

If $P(x)$ and $D(x)$ are polynomials with $D(x) \neq 0$ then there are unique polynomials $Q(x)$ and $R(x)$ such that

$$P(x) = D(x) \cdot Q(x) + R(x) \qquad (1)$$

where P, D, Q, and R are the *dividend*, *divisor*, *quotient*, and *remainder*.

Example 2 Let $P(x) = 3x^4 - 5x^3 - 20x - 5$ and $D(x) = x^2 + x + 3$. Find polynomials $Q(x)$ and $R(x)$ such that $P(x) = D(x) \cdot Q(x) + R(x)$.

Recall that when the divisor is of the form $x - c$, long division can be simplified by using **synthetic division**.

Example 3 Use synthetic division to simplify $\frac{2x^4 - 5x^2 + 8x - 3}{x - 2}$.

The Remainder Theorem

If the polynomial $P(x)$ is divided by $x - c$ then the remainder is $P(c)$.

Proof:

Using equation (1) with $D(x) = x - c$, and evaluating $P(c)$ we have

$$\begin{aligned} P(x) &= (x - c) \cdot Q(x) + R(x) \\ P(c) &= (c - c) \cdot Q(c) + R(c) \\ P(c) &= 0 \cdot Q(c) + R(c) \\ P(c) &= R(c) \end{aligned}$$

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Example 4 Let $P(x) = 3x^4 - 2x^2 - 5x + 3$.

(a) Find the quotient and remainder when $p(x)$ is divided by $x + 2$.

(b) Use the Remainder Theorem to find $P(-2)$

Note that if the remainder is equal to zero when dividing $p(x)$ by $x - c$, then $x - c$ is a **factor** of $P(x)$. This leads to the Factor Theorem:

The Factor Theorem

c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

Example 5 Use the Factor Theorem to show that $x = 4$ is a zero of $P(x) = x^3 - 3x^2 - 10x + 24$. Find the remaining zeros and write P in factored form.

Example 6 Find a polynomial of degree 4 that has zeros $-4,\ 0,\ 2,$ and 3, and graph the polynomial on your calculator.

Example 7 Find a polynomial with **integer** coefficients and zeros $\frac{-2}{3}$, $\frac{5}{2}$, and -2.