2.7 Combining Functions

The Sum, Difference, Product, and Quotient of Functions

Let f and g be functions with domains A and B. The, $f + g$, $f - g$, $f \cdot g$, and f/g are defined as follows:

1.	$(f + g)(x) = f(x) + g(x)$	Domain $A \cap B$
2.	$(f - g)(x) = f(x) - g(x)$	Domain $A \cap B$
3.	$(f \cdot g)(x) = f(x) \cdot g(x)$	Domain $A \cap B$
4.	$(f/g)(x) = \dfrac{f(x)}{g(x)}$	Domain $\{x \in A \cap B \mid g(x) \neq 0\}$

Example 1 Given $f(x) = \sqrt{x+5}$ and $g(x) = -x + 2$ make a sketch of $(f + g)(x)$.

Example 2 For $f(x) = x^2 - 9$ and $g(x) = \frac{x+3}{x-2}$ simplify $(f/g)(x)$ and find the domain. Verify by graphing f/g.

Composite Functions

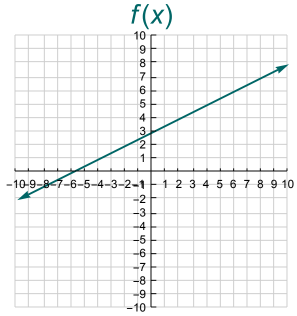
A composite function is when one function, f, is evaluated with another function, g, i.e., $f(g(x))$. The notation for a composite function is $(f \circ g)(x) = f(g(x))$.

Example 3 Given $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 6$ find and simplify $(f \circ g)(x)$. Evaluate $f(g(4))$ two different ways. (How are f and g related?)

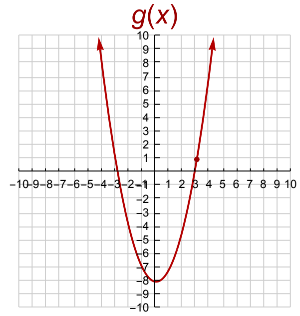
Example 4 Given the graphs of f and g, find the following expressions:

(a)	$(f \circ g)(2)$	(b)	$(f \circ g)(-4)$	(c)	$(g \circ f)(-2)$
(d)	$(f \circ f)(-8)$	(e)	$(g \circ g)(3)$		

$f(x)$



$g(x)$



Example 5 For *Example 4* above, find f and g, and

(a) find $(f \circ g)(x)$ and use it to verify parts (a) and (b) above

(b) find $(g \circ g)(x)$ and use it to verify part (e) above

Domain of a Composite Function

For a composite function $f(g(x))$, since the function g is evaluated first, the domain is initially restricted to that of g. In addition, since the **output** of g is used as input into f we need to make sure g's output does not give values that are restriction on the input for f.

For example, let $f(x) = \sqrt{x+3}$ and $g(x) = \frac{2}{x-4}$. We can see that for g, $x \neq 4$. Also, for f we have the input must be $x \geq -3$. Therefore, the output of g must be greater than or equal to -3, or $\frac{2}{x-4} \geq -3$. Solving this Inequality we have the domain for $(f \circ g)(x)$ as $\left(-\infty, \frac{10}{3}\right] \cup (4, \infty)$.

Example 6 Verify the domain of $f \circ g$ by graphing $(f \circ g)(x)$ where $f(x) = \sqrt{x-1}$ and $g(x) = \frac{5}{x-4}$.

Example 7 Find the domain for $f \circ g$ given $f(x) = \frac{2}{x-3}$ and $g(x) = x^2 + 4x + 6$.

Example 8 Find the functions f, g, and h, such that $f(g(h(x))) = \sqrt{(2x+3)^3} + 5$.