2.6 Modeling with Functions

Objectives: Analyze real world situations involving quadratics;

Example 1 A projectile is shot from the top of a 250 foot hill. Its height after t seconds is given by $h(t) = -16t^2 + 304t + 250$. Find the maximum height of the projectile, and the time it takes to hit the ground. (Recall the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.)

Example 2 A farmer wishes to build a corral using the side of the barn as one side of the corral. The corral is also to be divided down the middle to form two separate areas. Find the dimensions of the corral that maximizes the overall area assuming a maximum of 600 ft of fencing is used.

Example 3 A rectangle in quadrant 1 is bounded by the function $f(x) = \frac{-1}{2}x + 9$. Find the dimensions of the rectangle with maximum area, and find that maximum area.

Example 4 A rectangular piece of paper measures 15 cm by 20 cm. Squares are cut out of each corner, and the sides folded up to make a box without a top. Find the dimension of the squares that will maximize the volume, and the maximum volume.

Example 5 A rectangle in the first quadrant is bounded by $f(x) = 9 - x^2$. Find the dimensions of the rectangle with greatest area, and find the area.

Example 6 Find a function that models the distance from the point (1, 3) to the curve $f(x) = x^2$. Graph the distance function and find the point on the curve that is closest to (1, 3).

Challenge A 100 cm wire is cut into two pieces. One piece is bent into a circle, and the other is bent into a square. Find the point x where the wire should be cut to: (a) minimize the total area, and (b) maximize the total area.