

2.5 Quadratic Functions

Objectives: Identify a quadratic using transformations; write a quadratic in vertex form and general form; find the vertex of a quadratic; graph a quadratic; find the equation of a quadratic given the vertex and one other point.

A **quadratic function** is any function that can be written

$$f(x) = ax^2 + bx + c \text{ where } a \neq 0$$

The graph of a quadratic is a **parabola**, the simplest being the toolkit function $f(x) = x^2$. The *vertex* of the parabola is at $(0, 0)$, and the *line of symmetry* is the vertical line $x = 0$. (Figure 1)

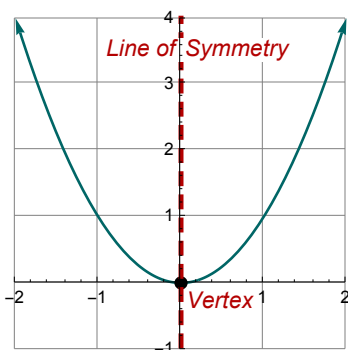


Figure 1

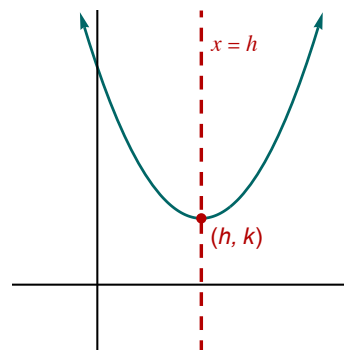


Figure 2

Vertex Form

By applying a right shift of h units, a vertical shift of k units, and a vertical scaling factor of a , we get the equation of a quadratic in **vertex form**:

$$f(x) = a(x - h)^2 + k$$

Here, the vertex is at (h, k) and the line of symmetry is $x = h$. (Figure 2.5.2)

Example 1 Make a sketch of the quadratic $f(x) = -2(x - 3)^2 + 8$, labeling the vertex, line of symmetry, and the x and y intercepts.

Vertex Formula for the General Quadratic

If we expand the vertex form of the quadratic, and compare it to the general form, we can derive a formula for the x coordinate of the vertex:

$$a(x - h)^2 + k = ax^2 - 2ahx + ah^2 + k$$

And so,

$$ax^2 - 2ahx + ah^2 + k = ax^2 + bx + c$$

which means $-2ah = b$. Solving for h (the x coordinate of the vertex) we have

$$\text{The } x \text{ and } y \text{ coordinates of the vertex are: } h = \frac{-b}{2a}, \text{ and } k = f\left(\frac{-b}{2a}\right).$$

Example 2 Find the vertex of the parabola $f(x) = -0.25x^2 + 3.5x - 2$.

Example 3 Write the equation of the parabola in vertex form: $f(x) = -2x^2 - 12x - 11$.

Example 4 Complete the square using the function from Example (3) to convert from general form to vertex form.

Example 5 Find the equation of the parabola with vertex $(2, -3)$ and passes through the point $(6, 4)$.

Example 6 Use your calculator's **QuadReg** function to find the equation of a parabola passing through the points $(2, 5)$, $(3, 2)$, and $(7, 10)$, and write the parabola in vertex form.