

## 2.4 Transformations of Functions

**Objectives:** Graph functions using vertical and horizontal shifts, vertical and horizontal scalings, and vertical and horizontal reflections.

### Vertical Shift

💡 A **vertical shift** is when a constant is added or subtracted from the entire *function*:  $g(x) = f(x) + k$ . This transforms the point  $(x, y) \rightarrow (x, y + k)$ .

**Example 1** Make a sketch of the function  $f(x) = \sqrt{x} + 3$

**Example 2** Given  $f(x) = |x|$ , make a sketch of  $g(x) = f(x) - 4$

### Horizontal Shift

💡 A **horizontal shift** occurs when a constant is added or subtracted from the *variable*:  $g(x) = f(x + k)$ . This transforms the point  $(x, y) \rightarrow (x - k, y)$ .

**Example 3** Sketch the function  $f(x) = \sqrt{x - 2} - 3$

### Vertical Scaling (Stretch or Compression)

💡 **Vertical scaling** occurs when the entire *function* is multiplied by a constant:  $g(x) = c f(x)$ . This transforms the point  $(x, y) \rightarrow (x, c y)$ .

**Example 4** Given  $f(x) = |x| - 2$ , make a sketch of  $g(x) = 3 f(x)$

## Horizontal Scaling

💡 **Horizontal scaling** occurs when the *variable* is multiplied by a constant:  $g(x) = f(cx)$ . This transforms the point  $(x, y) \rightarrow \left(\frac{1}{c}x, y\right)$

**Example 5** Identify the transformations used on the toolkit function  $f(x) = \llbracket x \rrbracket$  to get  $g(x) = \llbracket \frac{x}{3} \rrbracket + 2$  and sketch the function.

**Example 6** Identify the transformations used on the toolkit function  $f(x) = \sqrt{x}$  to get  $g(x) = \frac{1}{2}\sqrt{3x+12} + 1$ .

## Vertical and Horizontal Reflections

💡 A vertical **reflection** occurs when multiplying a function by  $-1$ ,  $g(x) = -f(x)$ , transforming the point  $(x, y) \rightarrow (x, -y)$ . A **horizontal reflection** is obtained by multiplying the variable by  $-1$ ,  $g(x) = f(-x)$ , transforming the point  $(x, y) \rightarrow (-x, y)$ .

**Example 7** Use transformations to make a sketch of the function  $f(x) = -2(x-3)^2$ .

💡 **Example 8** Identify all of the transformations applied to  $f(x) = x^3$  to obtain  $g(x) = -\left(\frac{1}{2}x - 2\right)^3 + 3$ , and make a sketch of  $f(x)$ .

**Example 9** If the point  $(6, -5)$  is on the graph of  $y = f(x)$ , where is the corresponding point on the graph of  $y = -3f(2x + 10) + 7$ ? Show the original point under each transformation.