2.3 Properties of Functions

Objectives: Determine if a function is odd even or neither; find the local and global extrema of a function; find the intervals a function is increasing or decreasing; find the average rate of change of a function.

Even and Odd Functions

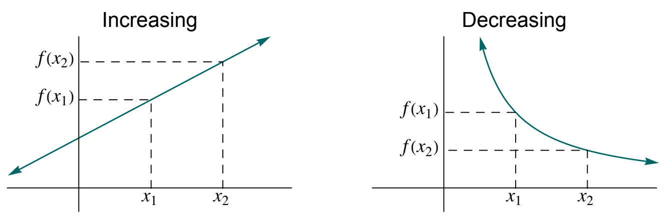
- An **even** function is such that $f(-x) = f(x)$, and is symmetric with respect to the y axis.
- An **odd** function is such that $f(-x) = -f(x)$, and is symmetric with respect to the *origin*.

Example 1 Determine if the following function is even, odd, or neither: $f(x) = -2x^4 + 6x^2 + 3$.

Example 2 Determine if the following function is even, odd, or neither: $f(x) = \frac{x^3 - 9x}{x^2 + 1}$.

Increasing and Decreasing Functions

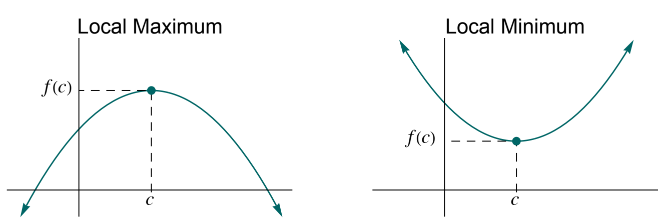
- A function is **increasing** on an open interval (a, b) if $f(x_2) > f(x_1)$ whenever $x_2 > x_1$ in (a, b).
- A function is **decreasing** on an open interval (a, b) if $f(x_2) < f(x_1)$ whenever $x_2 > x_1$ in (a, b).



Example 3 Graph the function $f(x) = x^3 - 8x^2 + 16x - 3$ on your calculator and determine the intervals on which the function is increasing, and where it is decreasing.

Local Maximum Values and Local Minimum Values

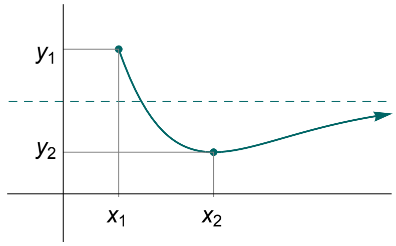
A local max (or min) value is when a function value is greater (or smaller) than any other value "near" it on an open interval.



Example 4 Find the local extrema for the function in Example 3.

Absolute Maximum Values and Absolute Minimum Values

The absolute max (or min) value is the overall largest (or smallest) value a function has on the entire domain of the function



Example 5 Find the absolute extrema of the function $f(x) = x^2 - 6x + 11$ on the closed interval [1, 4].

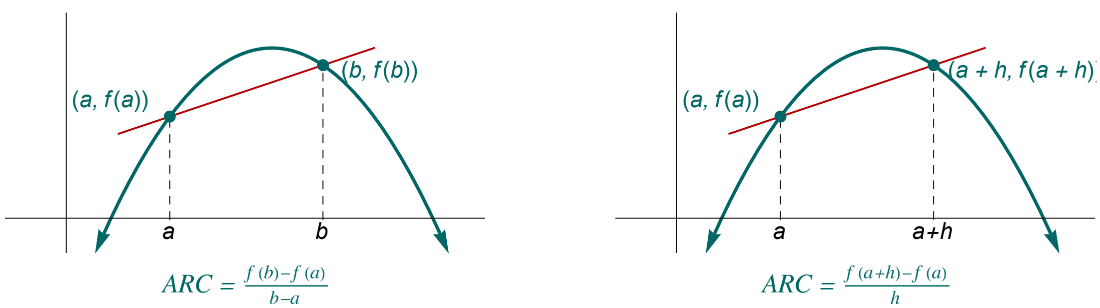
Average Rate of a Function

The *average rate of change of* $f(x)$ from $x = a$ to $x = b$ is defined as

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

Example 6 Find the average rate of change of $f(x) = 3x^2 - 3x + 8$ from 1 to 3.

The average rate of change of a function can be interpreted as the slope of the *secant line* of a function through two points, and has two different forms depending on how the graph is labeled:



Example 7 Find the slope of the secant line for $f(x) = x^2$ on the intervals [2, 4]; [2, 3]; [2, 2.1]; [2, 2.0001]. What value is the secant slope approaching?