

2.1 What is a Function

Objectives: Identify the graph of a function; obtain information about or from the graph of a function; simplify the difference quotient given a function.

A function is a relation that assigns exactly one value, y , in the range to each x value in the domain. x is often referred to as the *independent* variable, and y is the *dependent* variable.

Example 1 Which of the following are functions? Those that are functions, identify the independent and dependent variable:

- (a) $y = 2x + 5$
- (b) John's paycheck amount and the number of hours he worked.
- (c) The set of points $\{(3, 6), (4, 2), (5, 2), (3, 1)\}$.

Example 2 For the function $f(x) = x^2 - 3x + 8$, find

- (a) $f(3)$
- (b) $f(-2)$
- (c) $f(x + 1)$
- (d) $f(2a)$

Simplifying the *Difference Quotient*

The difference quotient is the expression: $\frac{f(x+h)-f(x)}{h}$, and is used repeatedly in calculus.

Example 3 Simplify the difference quotient using the function $f(x) = 2x^2 - 4x - 3$.

Example 4 Simplify the difference quotient using the function $f(x) = \frac{2}{x+3}$.

Piecewise Functions

Example 5 Evaluate the piecewise function $f(x) = \begin{cases} x^2 - 3x & \text{if } x < -2 \\ 3 & \text{if } -2 \leq x < 3 \\ x - 1 & \text{if } x \geq 3 \end{cases}$.

(a) $x = -4$

(b) $x = -2$

(c) $x = 0$

(d) $x = 4$

Domain and Range of a Function

💡 The domain is the set of all possible x values a function can be evaluated at (the input), and the range is the set of all possible y values from the function (the output).

Example 6 Find the domain of the functions. Write the domains using interval notation, and graph the function on your calculator.

(a) $f(x) = \frac{x-7}{x^2-5x-6}$

(b) $g(x) = \frac{x}{\sqrt[3]{x-1}}$

(c) $h(x) = \frac{\sqrt{x^2-3x}}{x-5}$