

1.7 Solving Inequalities

Objectives: Solve polynomial inequalities using sign diagrams and graphs; Solve rational inequalities using sign diagrams and graphs; Solve absolute value inequalities.

💡 The most efficient way to solve a polynomial or rational inequality is to use a *sign diagram*. First, be sure that the inequality is set to zero. Next, find the zeros for all factors, and finally create a sign diagram to solve the inequality.

Example 1 Use a *sign diagram* to solve the inequality $3x^2 + 2x - 21 \leq 0$.

Example 2 Solve the inequality $\frac{x^2 + 4x - 5}{x + 2} \leq 0$.

Example 3 Solve the inequality: $x^3 + 16 > 4x^2 + 4x$

Example 4 Solve the inequality: $\frac{x}{x-2} < \frac{2}{x-3}$

Example 5 Solve the inequality: $\frac{x^2-4x-5}{2x^2-7x-15} \geq 0$

Solving Absolute Value Inequalities

Properties of Absolute Value Inequalities

Inequality	Equivalent Form	Graph
1. $ x < c$	$-c < x < c$	
2. $ x \leq c$	$-c \leq x \leq c$	
3. $ x > c$	$x < -c$ or $x > c$	
4. $ x \geq c$	$x \leq -c$ or $x \geq c$	

Example 6 Solve the inequality: $3 + |2x + 4| \geq 9$

Example 7 Solve the inequality: $\left| \frac{3x-1}{5} \right| < 1$

Challenge Solve the inequality: $|3x - 2| \leq |x + 3| + 3$