Math 141 Pre-Calc I In-Class Notes

1.2 Review of Exponents and Radicals

Objectives: Use the properties of exponents; use rational exponents; use scientific notation; simplify radicals.

Exponential Notation

Integer exponents are used for repeated multiplication: $a^n = \underbrace{a \cdot a \cdots a}_{n \text{ factors}}$, a is the **base** and n is the **exponent**.

Laws of Exponents

1. $a^0 = 1$

2. $a^{-n} = \frac{1}{a^n}$

3. $a^m a^n =$

4. $\frac{a^m}{a^n} =$

5. $(a^m)^n =$

6. $(a\,b)^n =$

7. $\left(\frac{a}{b}\right)^n =$

8. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

9. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

Example 1 Simplify: $\frac{(2\,x^2)^3\,3\,x^4}{(x^3)^5}$

Example 2 Simplify: $(3\,a\,b^2\,c)\left(\frac{2\,a^{-2}\,b}{c^3}\right)^{-2}$

Scientific Notation

Scientific notation is used to write extremely large numbers or extremely small numbers in a more useful form. For instance, the number of molecules in a cup of water is approximately 8,360,000,000,000,000,000,000,000; the mass of a neutron is 0.00000 00000 00000 00000 00000 016749 kg. In scientific notation these numbers are

$$8{,}360{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 8.36 \times 10^{24}$$
$$0.000000000000000000000000016749 = 1.6749 \times 10^{-27} \text{ kg}$$

Example 3 Let $a = 0.00000361$, $b = 2.43 \times 10^{12}$, and $c = 3.27 \times 10^{-18}$. **(a)** Write a in scientific notation and **(b)** calculate $\frac{b}{a\,c}$.

Radicals and n^{th} roots

If n is any positive integer, then the **principal n^{th}root of a** is defined as $\sqrt[n]{a} = b$ meaning $b^n = a$. If n is even then $a \geq 0$ and $b \geq 0$.

Properties of nth Roots

1. $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$
2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
4. $\sqrt[n]{a^n} = a$ if n is odd
5. $\sqrt[n]{a^n} = |a|$ if n is even

Example 4 Simplify: **(a)** $\sqrt[3]{-27x^6y^9}$ **(b)** $\sqrt[4]{16x^4y^8z^{16}}$

Example 5 Simplify: $x\sqrt{27x} + 2\sqrt{75x^3}$

Rational Exponents

By definition, $a^{1/n} = \sqrt[n]{a}$, for the rules of exponents and radicals to apply. In general, we have the following:

Definition of Rational Exponents

For any rational exponent m/n in lowest terms with $n > 0$, we define

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} \quad \text{If } n \text{ is even, then } a \geq 0$$

⌘ Calculator Tip: Since the cube-root of number can be written as a^$(1/3)$, and also $1/3 = 3^{-1}$, the cube root of 17 can be calculated as 17^3^{-1}. On your calculator, you may have an $\boxed{x^{-1}}$ button that can easily be used to calculate roots. For example, $\sqrt[5]{20}$ = 20^5 $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$.

Example 6 Solve the equations: **(a)** $x^4 = 80$ **(b)** $x^{2.432} = 20$

Example 7 Simplify: **(a)** $(2x^{3/2})(4x)^{-1/2}$ **(b)** $\left(\frac{a^2b^{-3}}{x^{-1}y^2}\right)\left(\frac{x^{-2}b^{-1}}{a^{3/2}y^{1/3}}\right)$ **(c)** $\sqrt[3]{x^2}\cdot\sqrt[4]{x^3}$